How Can We "Perfectly and Rapidly" Stitch Images? Exploring Improved End-to-end Techniques

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Before We Start

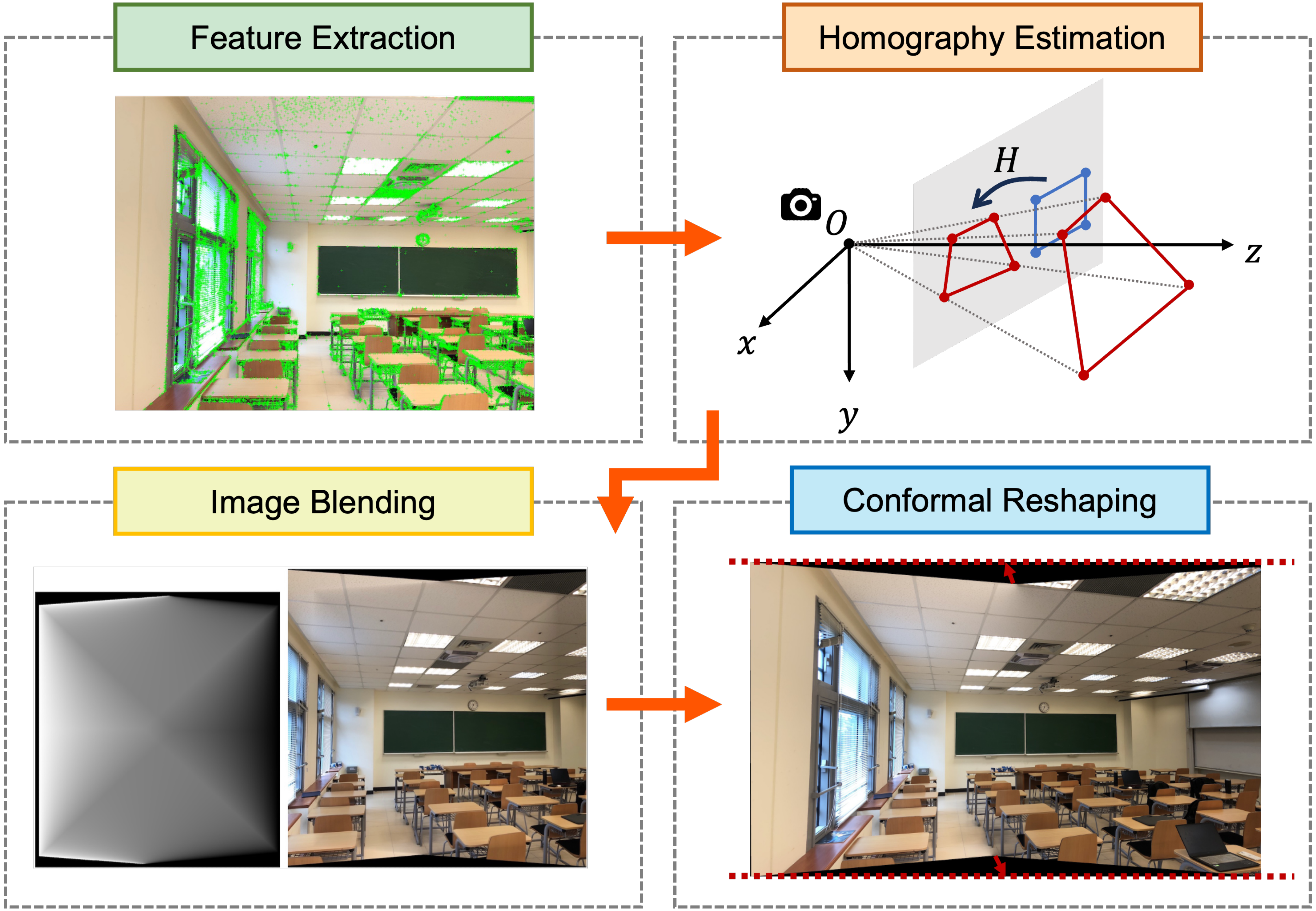
Slido

Slides

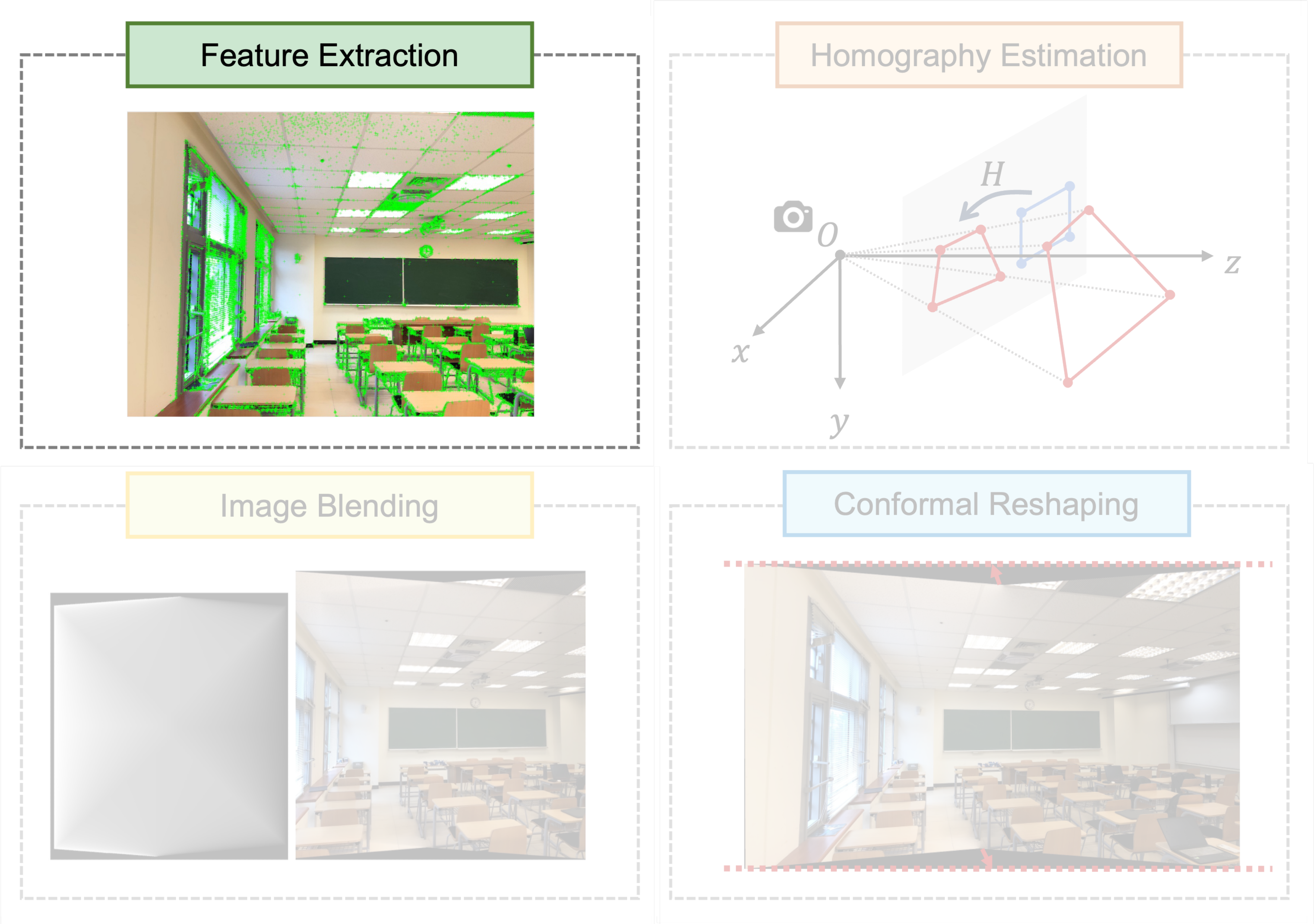
Introduction to Image Stitching

Case Study

End-to-End Image Stitching Pipeline



Feature Extraction



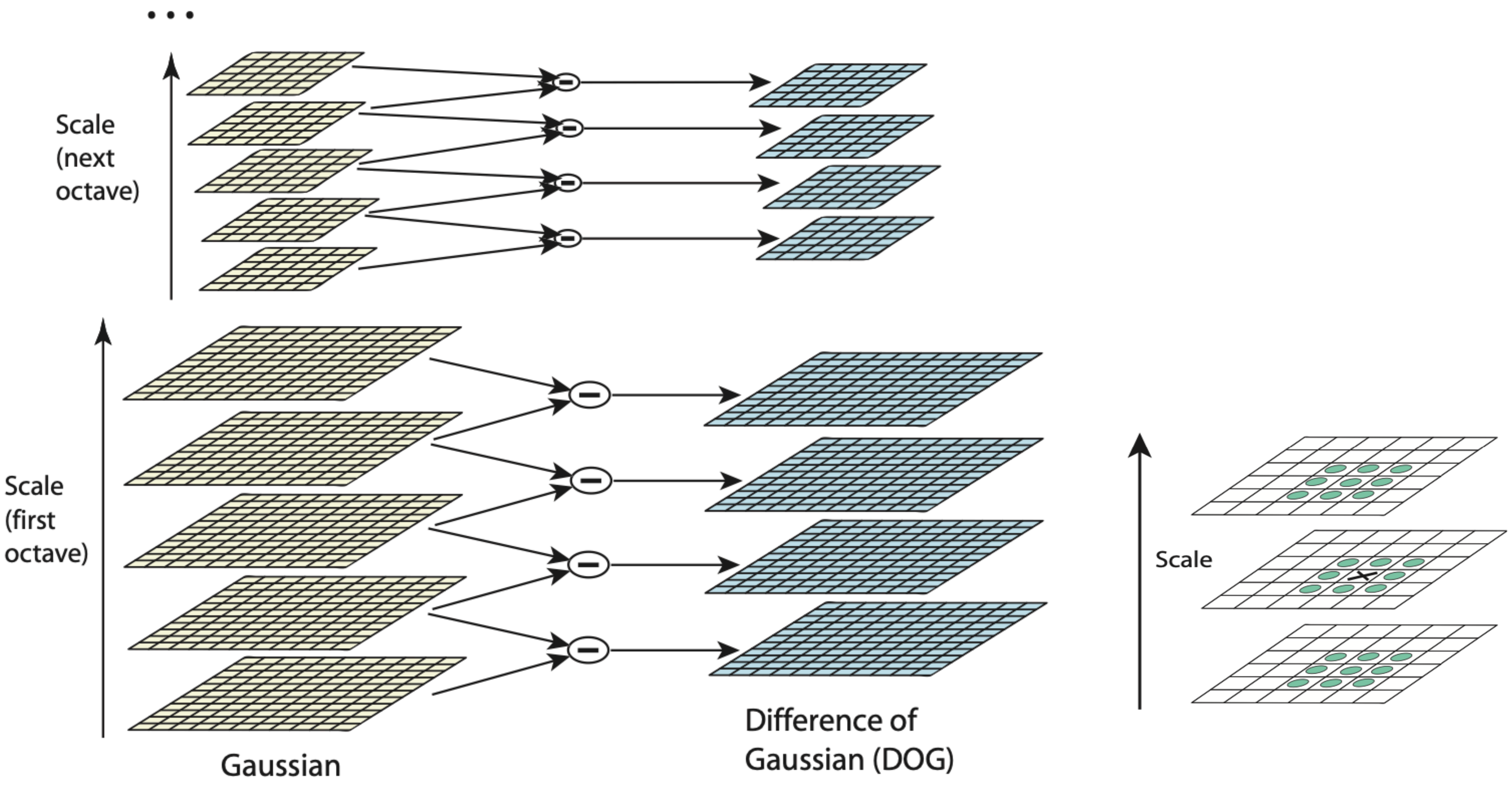
Feature Extraction: SIFT

The **S**cale-**I**nvariant **F**eature **T**ransform (SIFT)

Feature Extraction: SIFT

The **S**cale-**I**nvariant **F**eature **T**ransform (SIFT) has the following two steps:

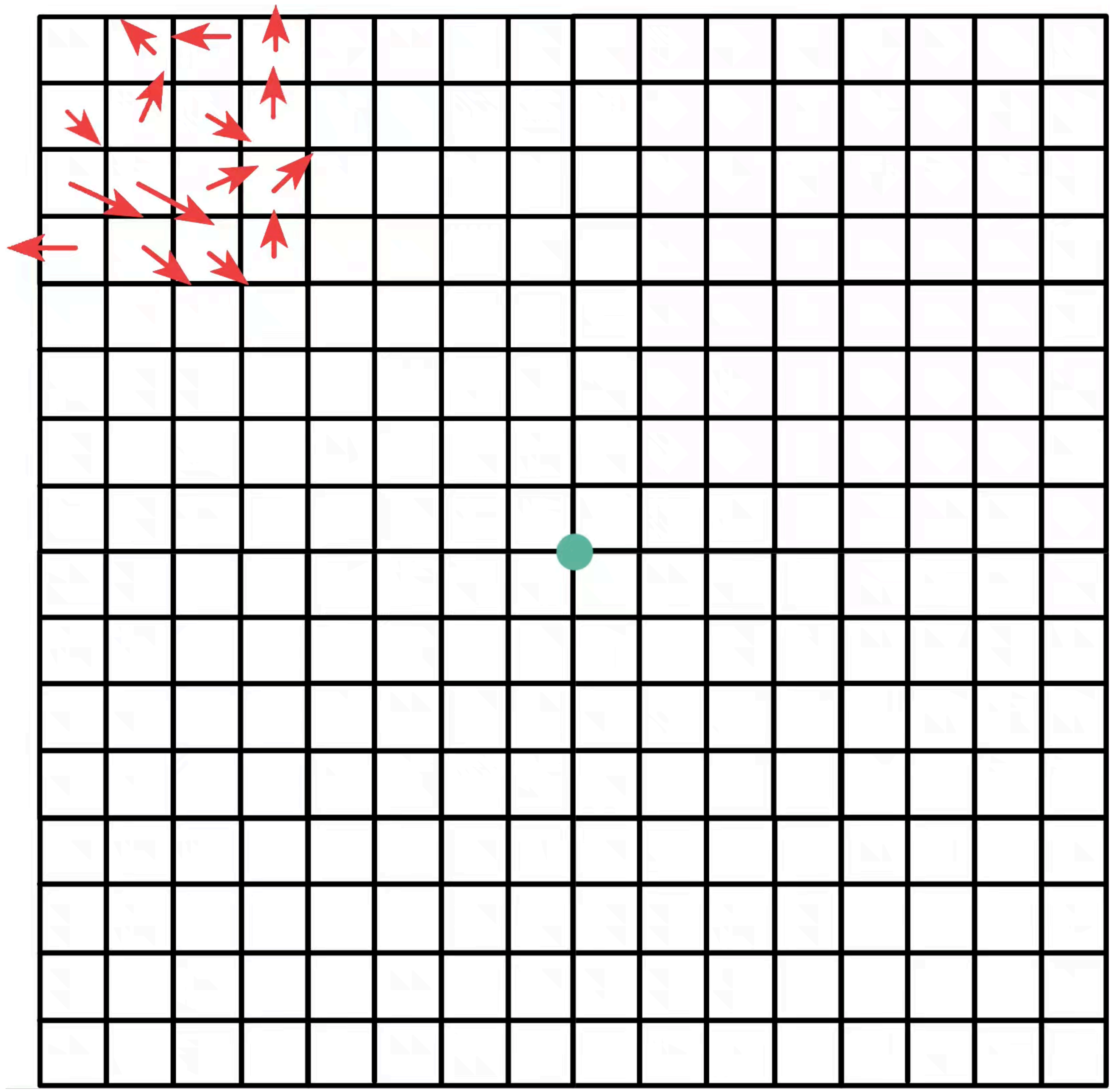
1. **Detection**: Extract key points.



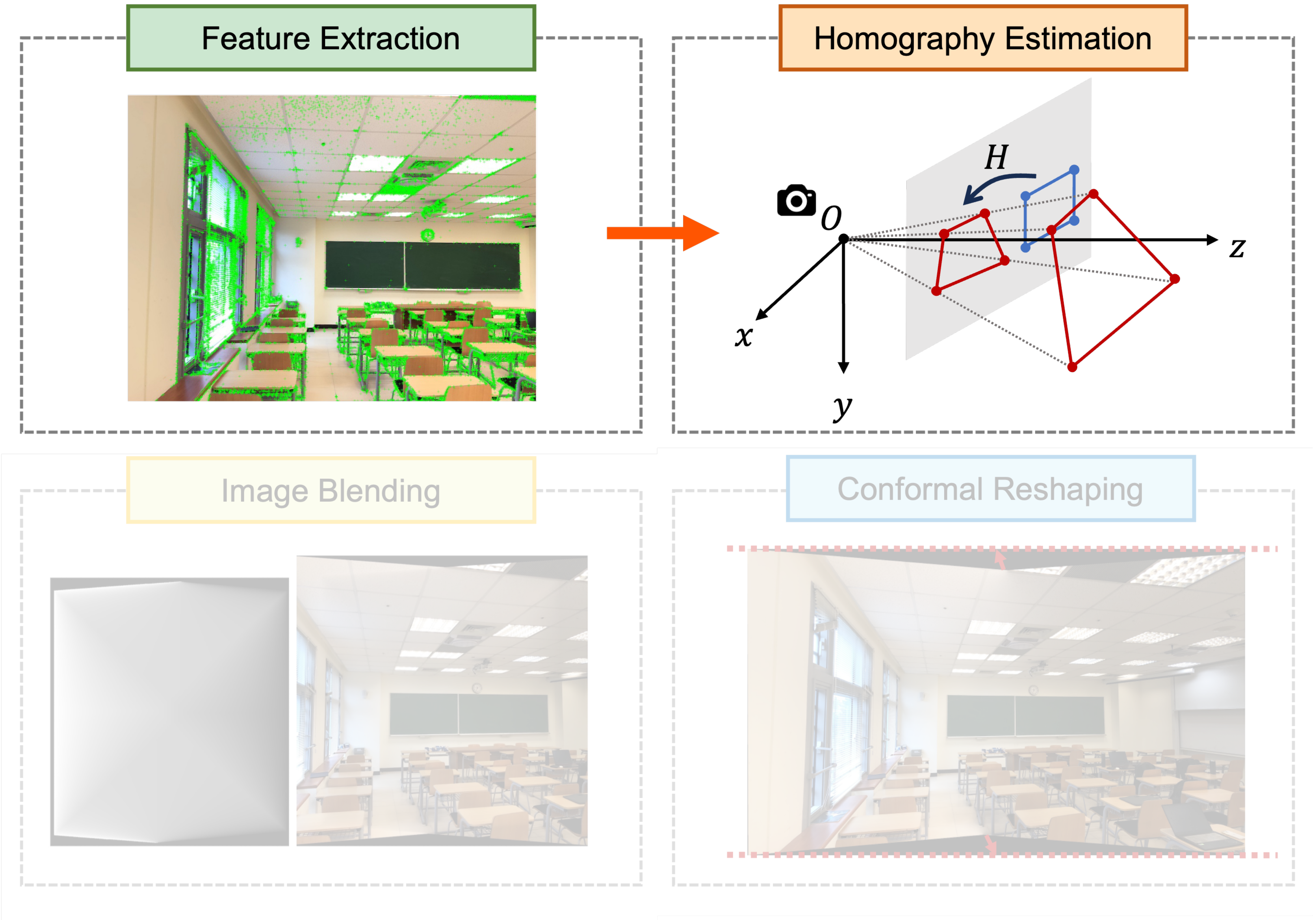
Feature Extraction: SIFT

The **S**cale-**I**nvariant **F**eature **T**ransform (SIFT) has the following two steps:

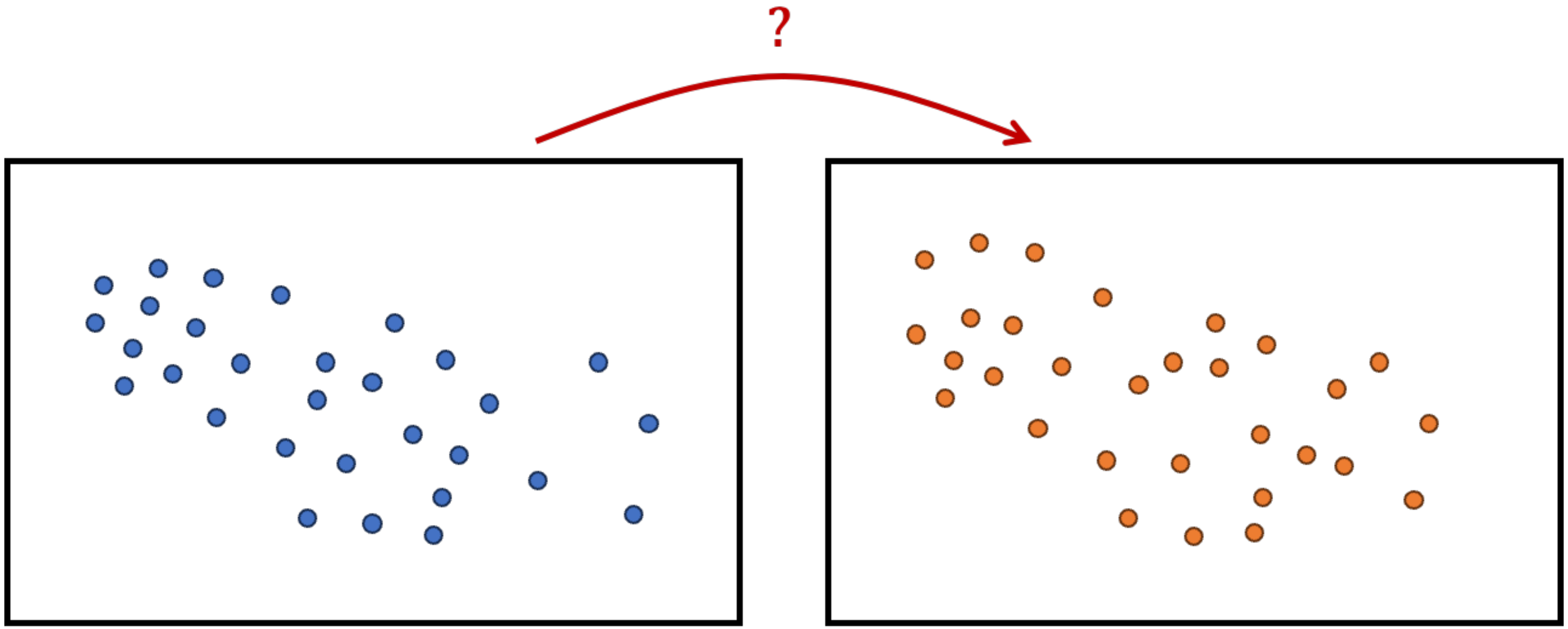
2. **Description**: Extract the feature vector for each key point.



Homography Estimation



Homography Estimation

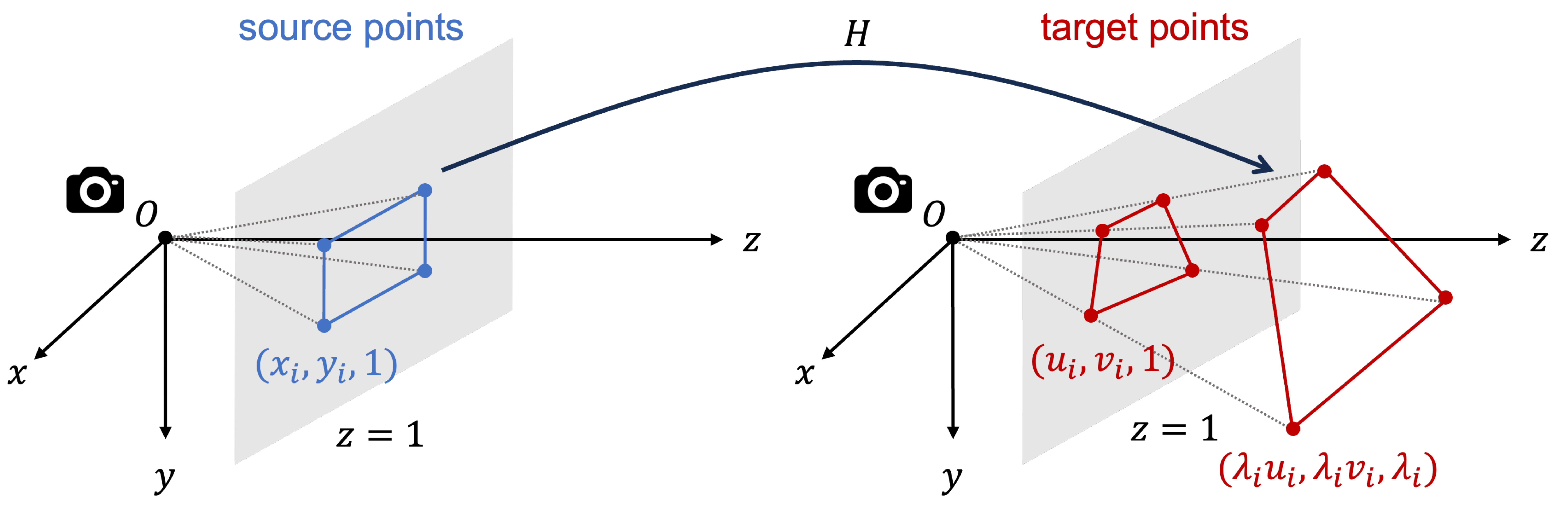


source points $\mathbf{p}_S$ target points $\mathbf{p}_T$

Homography Estimation: Camera Model

Given source points $\{(x_i, y_i)\}_{i=1}^n$ and target points $\{(u_i, v_i)\}_{i=1}^n$, the goal is to find the homography matrix mapping each source point to its corresponding target point.

$$\lambda_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} \\ h_{2,1} & h_{2,2} & h_{2,3} \\ h_{3,1} & h_{3,2} & h_{3,3} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}, \quad \lambda_i > 0, \quad i = 1, 2, \ldots, n$$



Homography Estimation: Linear System

We formulate homography as a $2n \times 9$ linear system:

$$
\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1 u_1 & -y_1 u_1 & -u_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1 v_1 & -y_1 v_1 & -v_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & y_n & 1 & 0 & 0 & 0 & -x_n u_n & -y_n u_n & -u_n \\
0 & 0 & 0 & x_n & y_n & 1 & -x_n v_n & -y_n v_n & -v_n
\end{bmatrix}
\begin{bmatrix}
h_{1,1} \\ h_{1,2} \\ h_{1,3} \\ h_{2,1} \\ h_{2,2} \\ h_{2,3} \\ h_{3,1} \\ h_{3,2} \\ h_{3,3}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ \vdots \\ 0 \\ 0
\end{bmatrix}
\implies A\mathbf{h} = \mathbf{0} \quad (1)
$$

Fact

- The degrees of freedom of $\mathbf{h}$ are 8, requiring at least 4 points.

Homography Estimation: Least Squares Problem

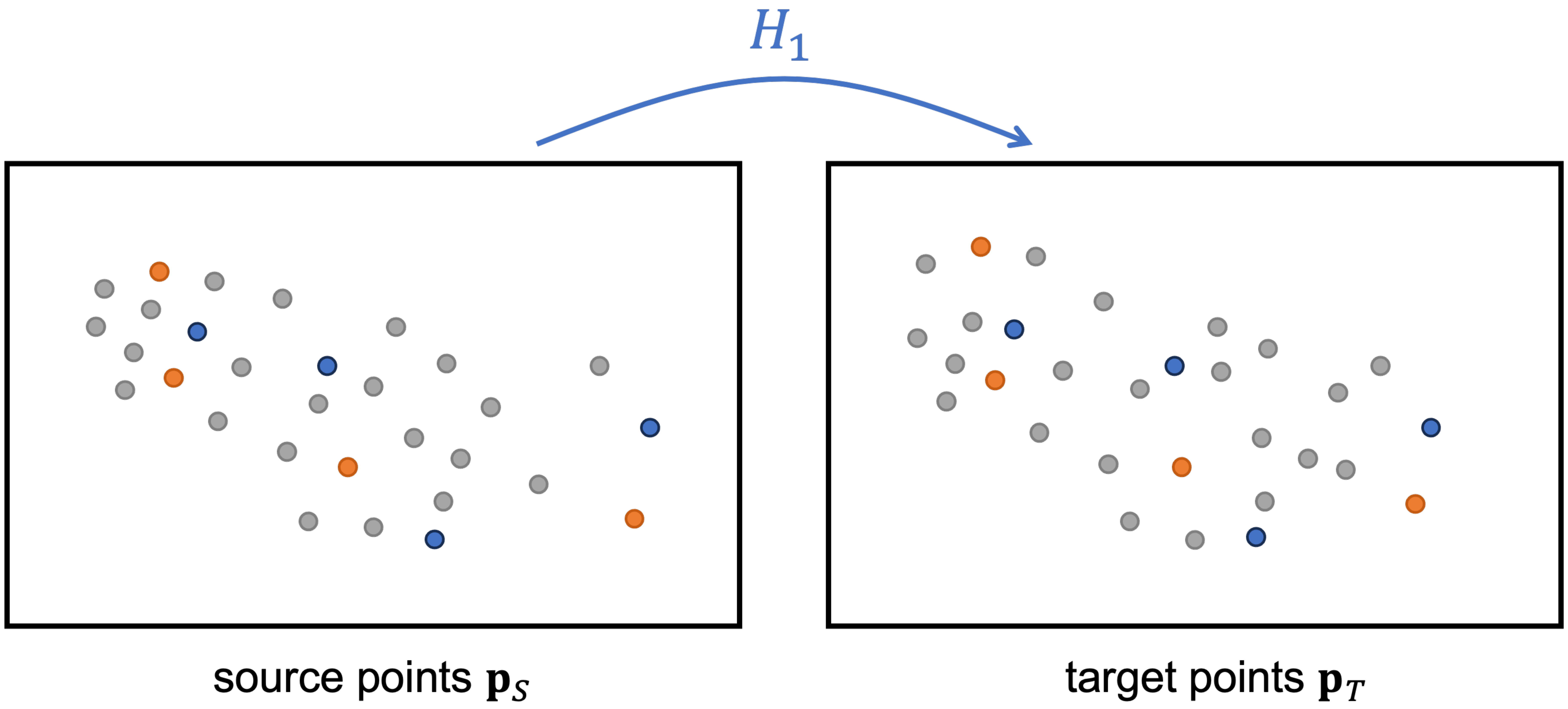
The linear system defined in (1) is reformulated as a least squares problem:

$$\min_{\|\mathbf{h}\|_2=1} \|A\mathbf{h} - \mathbf{0}\|_2^2. \tag{2}$$

Fact

- $\mathbf{h}$ is subject to arbitrary scaling, we incorporated the length constraint of 1 into the optimization formulation.
- The solution of (2) is the unit eigenvector of $A^\top A$ associated with its smallest eigenvalue.

Homography Estimation: RANSAC



Inlier error: $\|H_2\mathbf{p}_S - \mathbf{p}_T\|_2 > \|H_1\mathbf{p}_S - \mathbf{p}_T\|_2$

Stitching Result of Two Images

Figure: Stitching results of SIFT

Stitching Result of Two Images

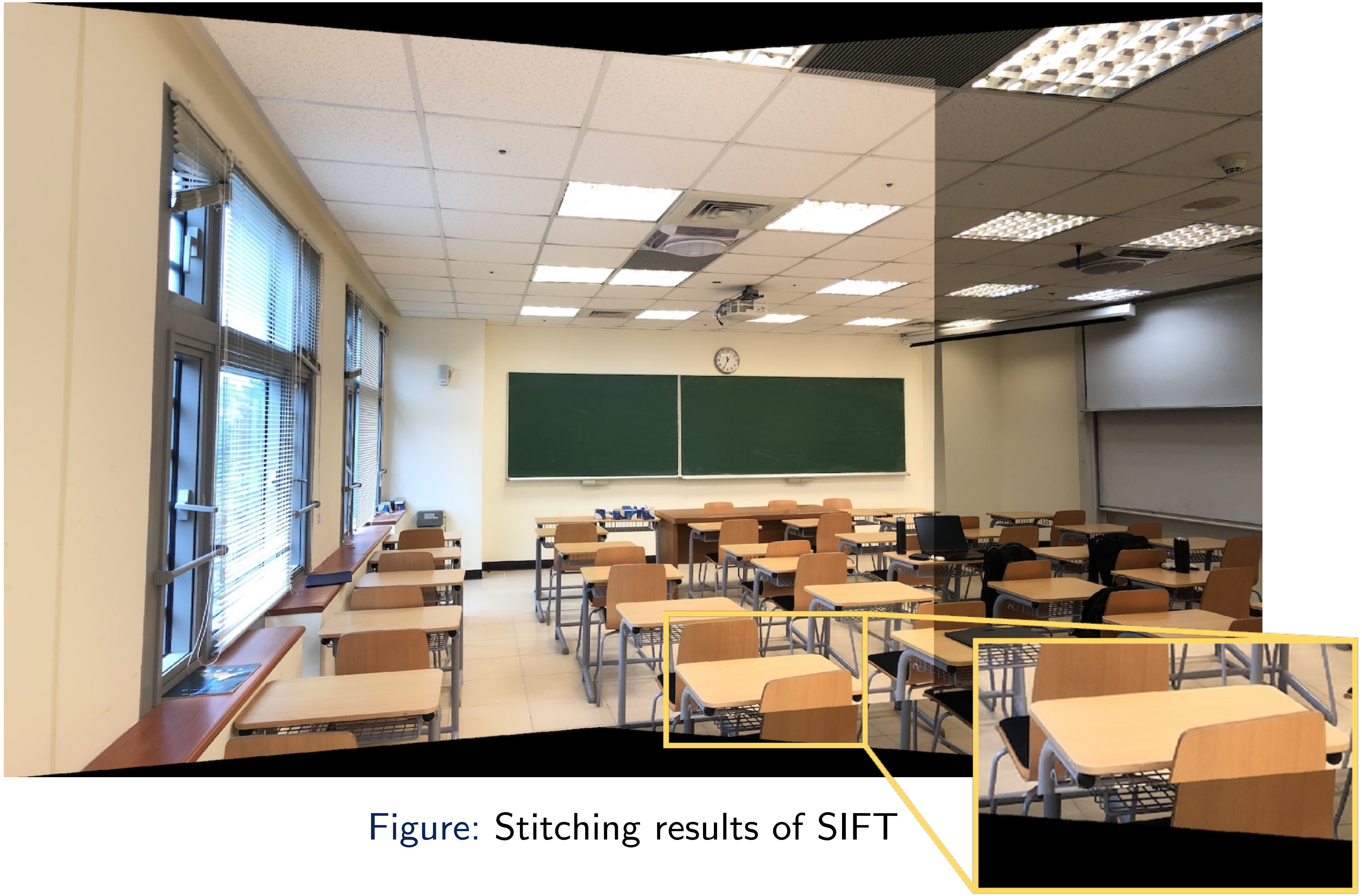


Figure: Stitching results of SIFT

Strategies to Improve Stitching Result

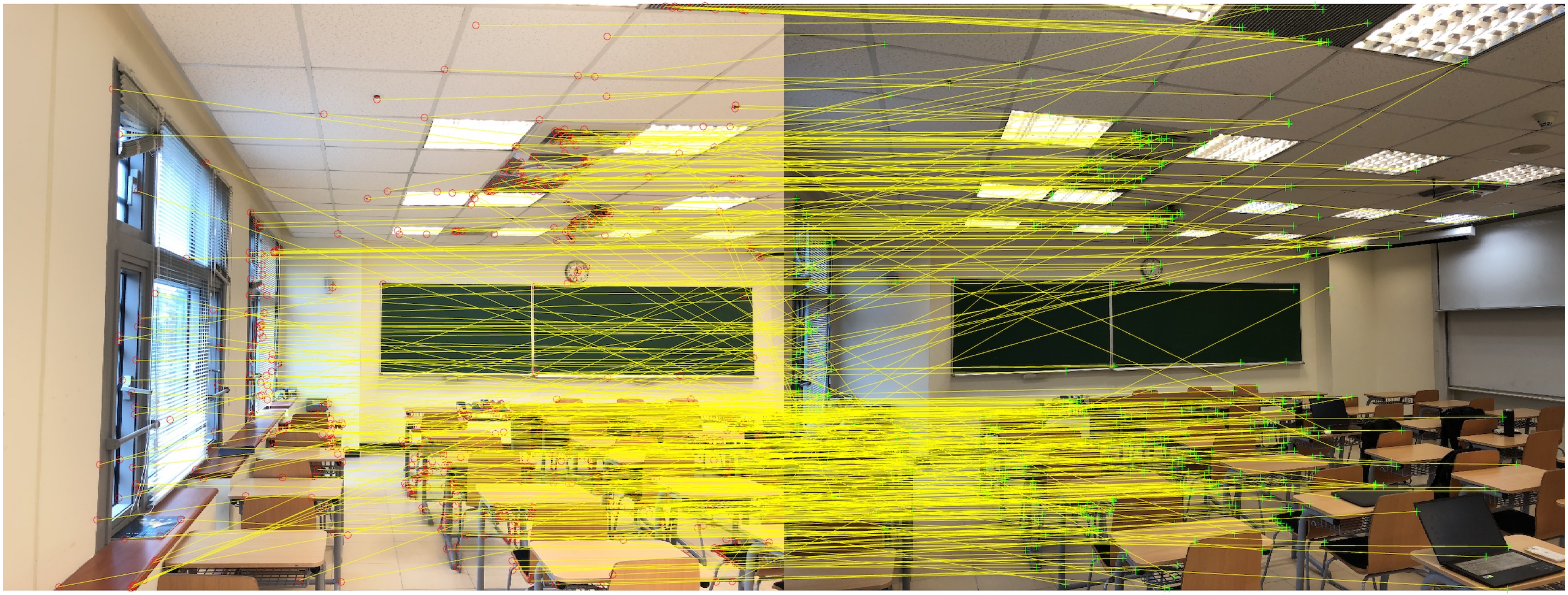
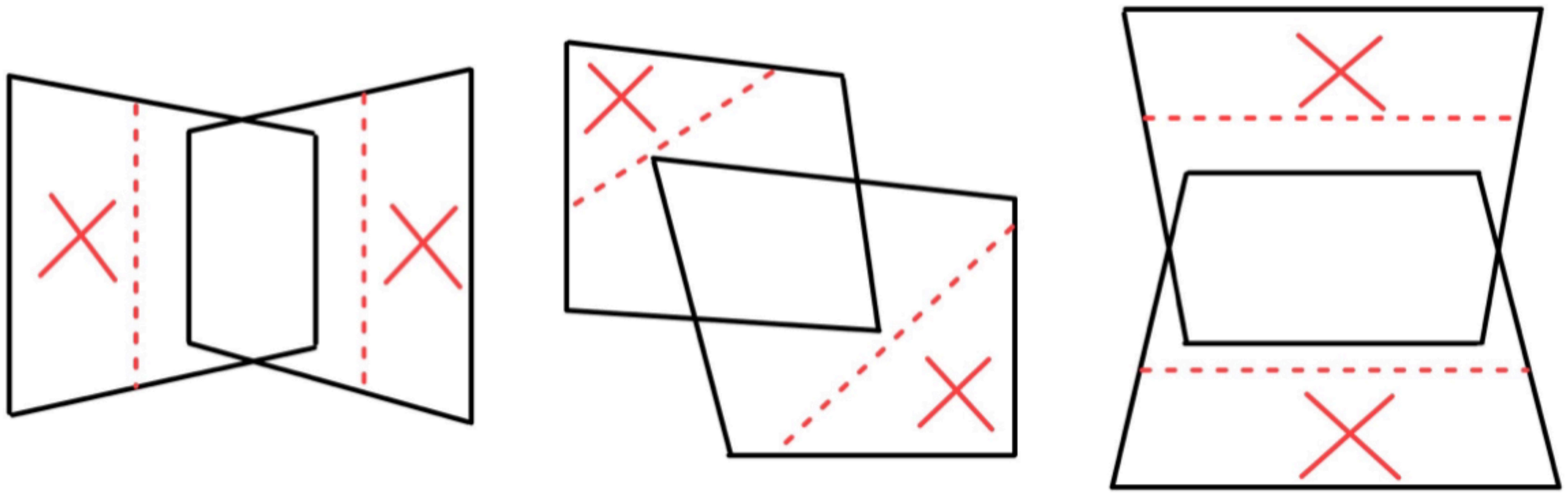


Figure: Matching results of SIFT

Strategies to Improve Stitching Result

1. Discard irrelevant points.



Strategies to Improve Stitching Result

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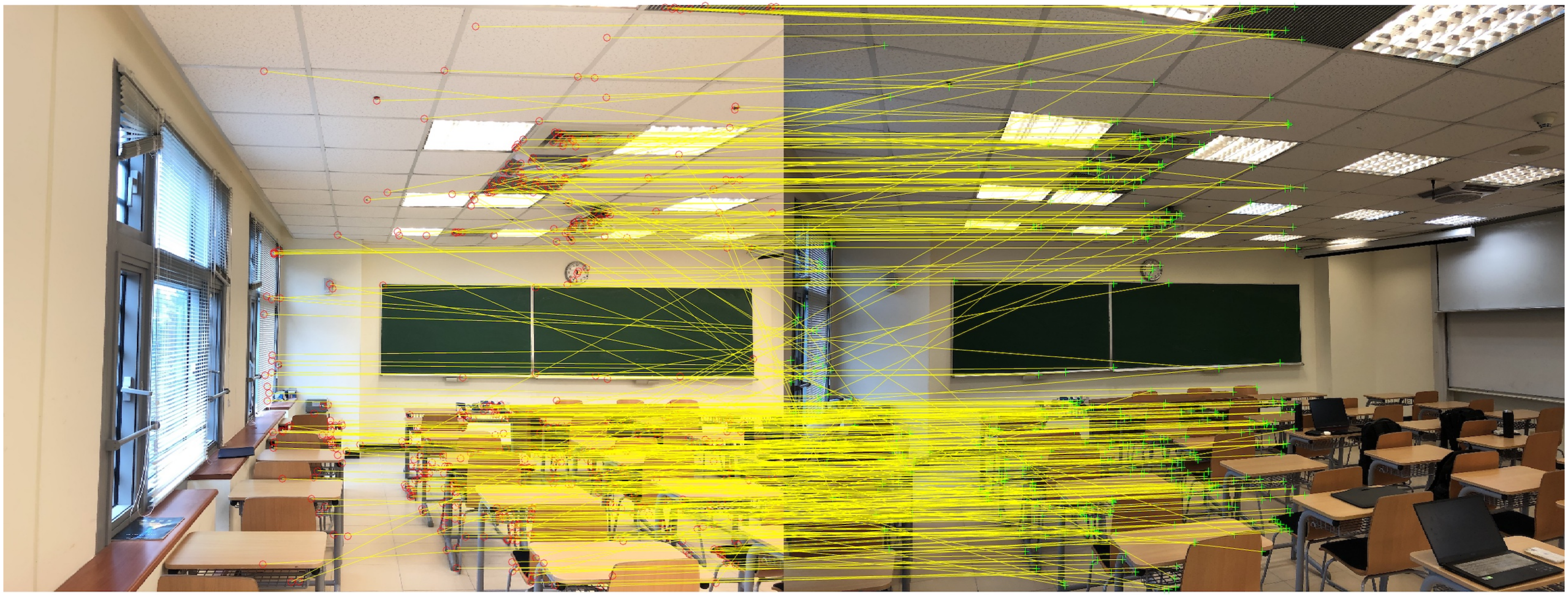
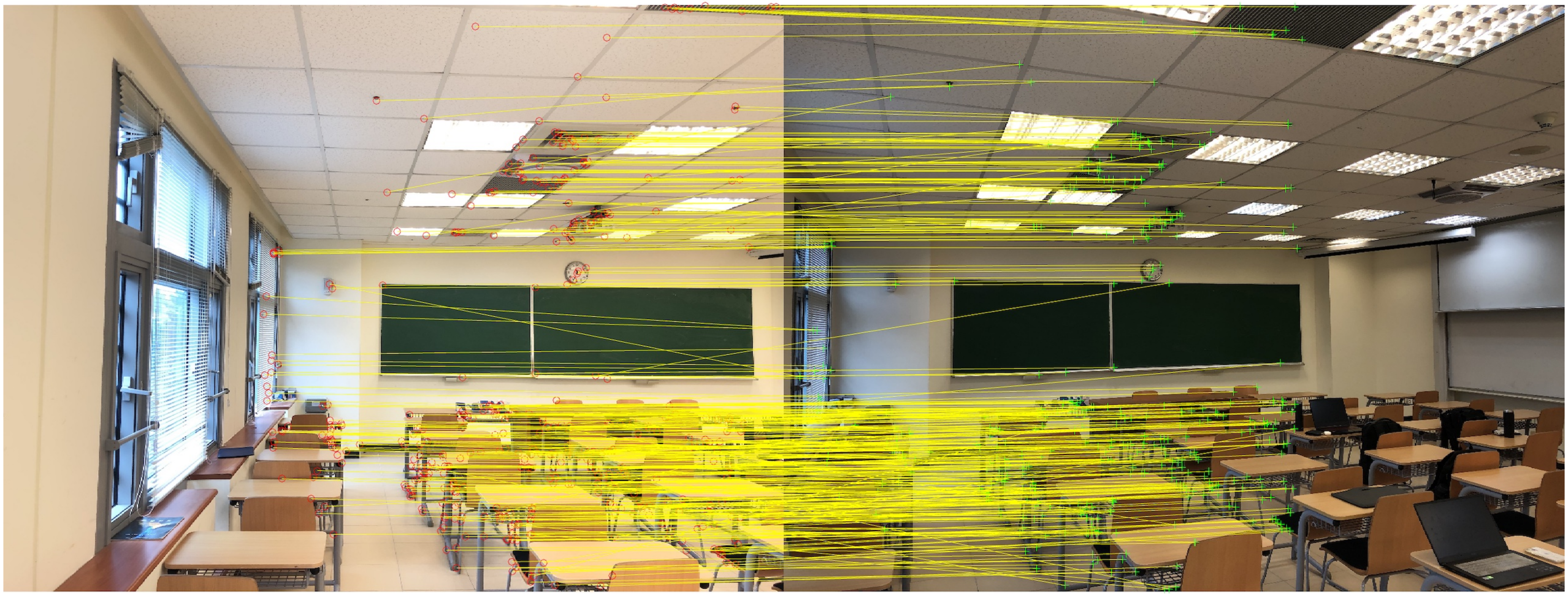


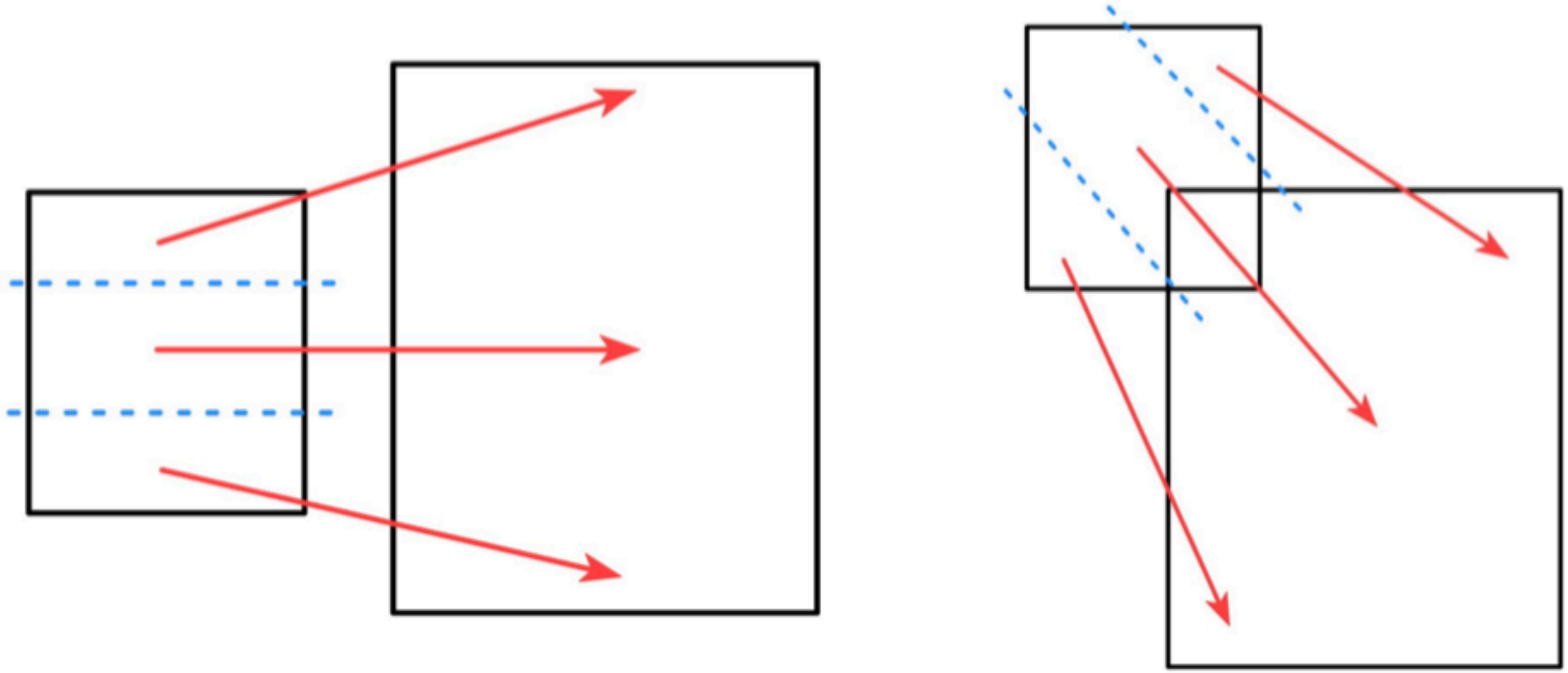
Strategies to Improve Stitching Result

2. Filter out pairings with excessive slope.



Strategies to Improve Stitching Result

3. Segment matched pairs into three parts and reapply slope filtering.



Strategies to Improve Stitching Result

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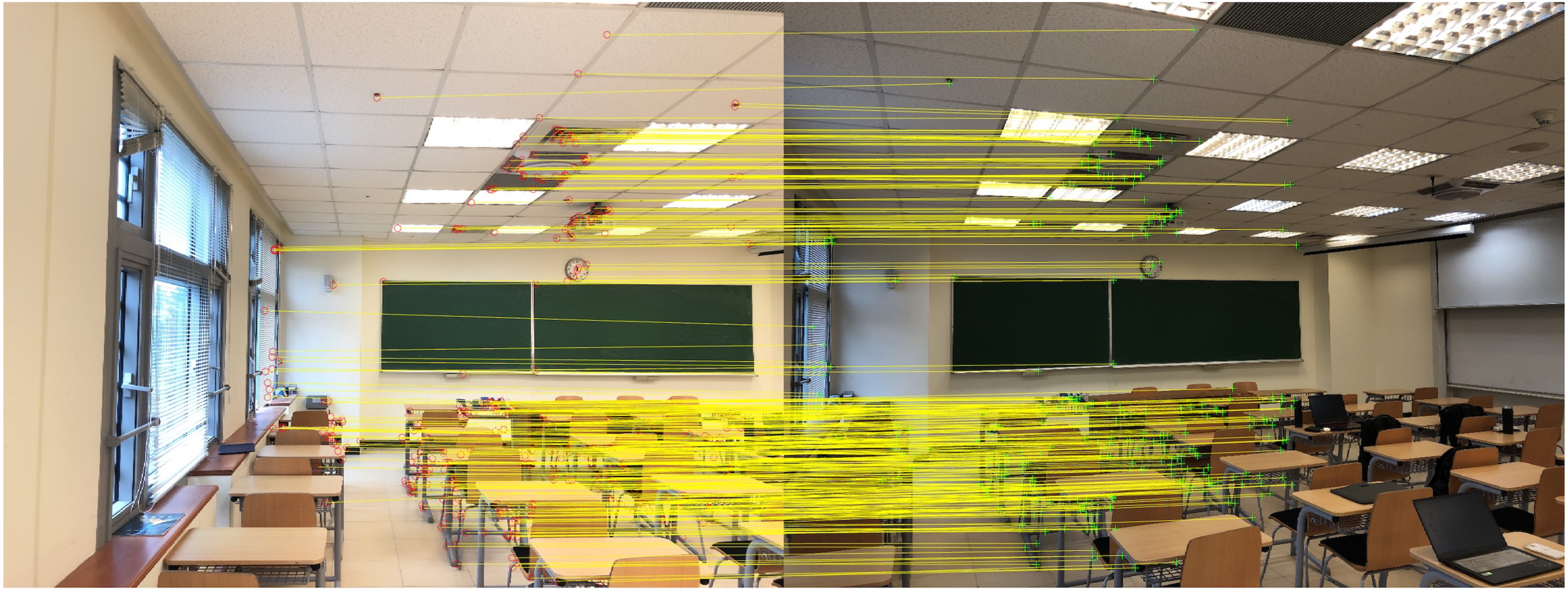


Strategies to Improve Stitching Result

Figure: Perfect matching results

Image Blending

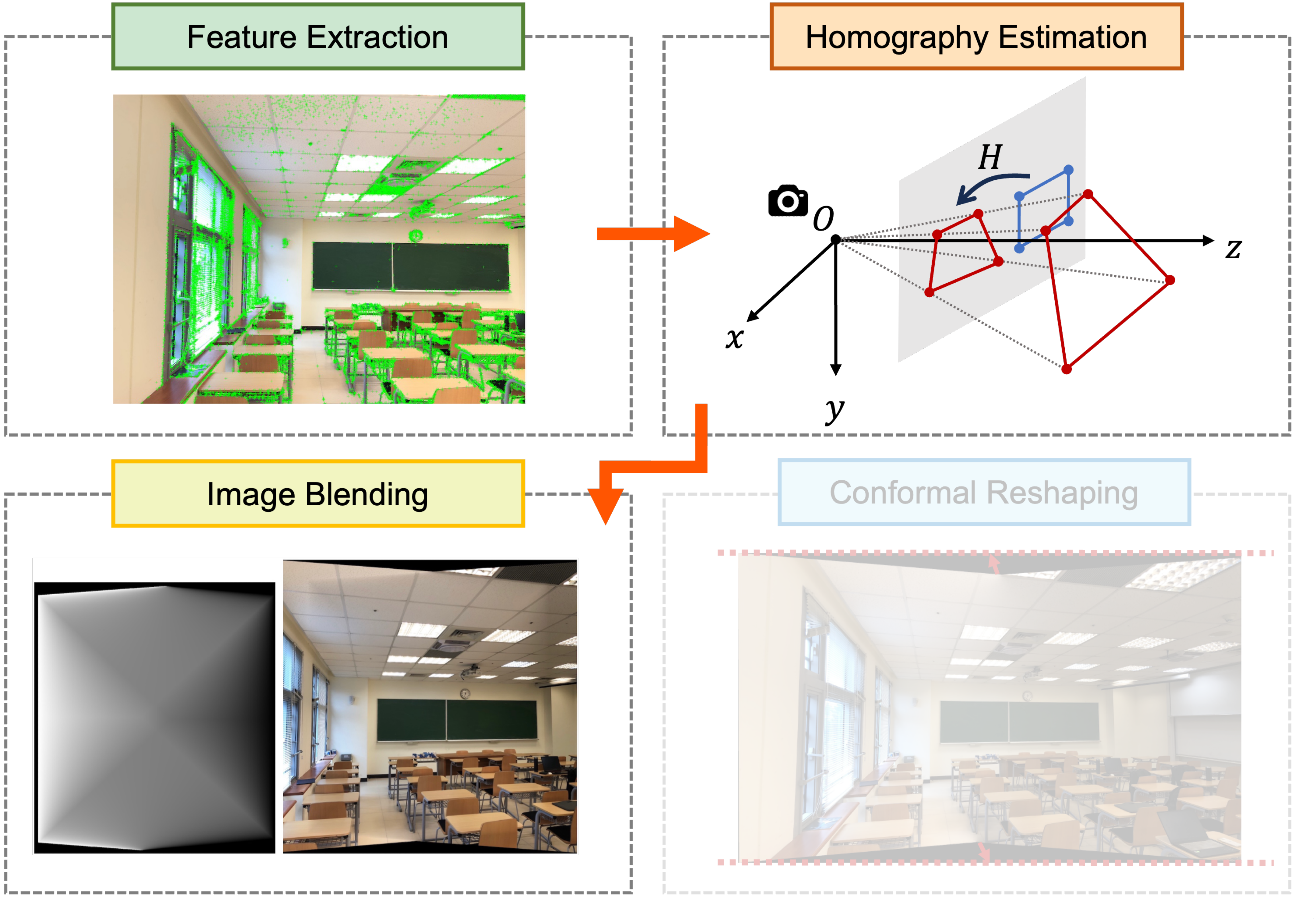


Image Blending

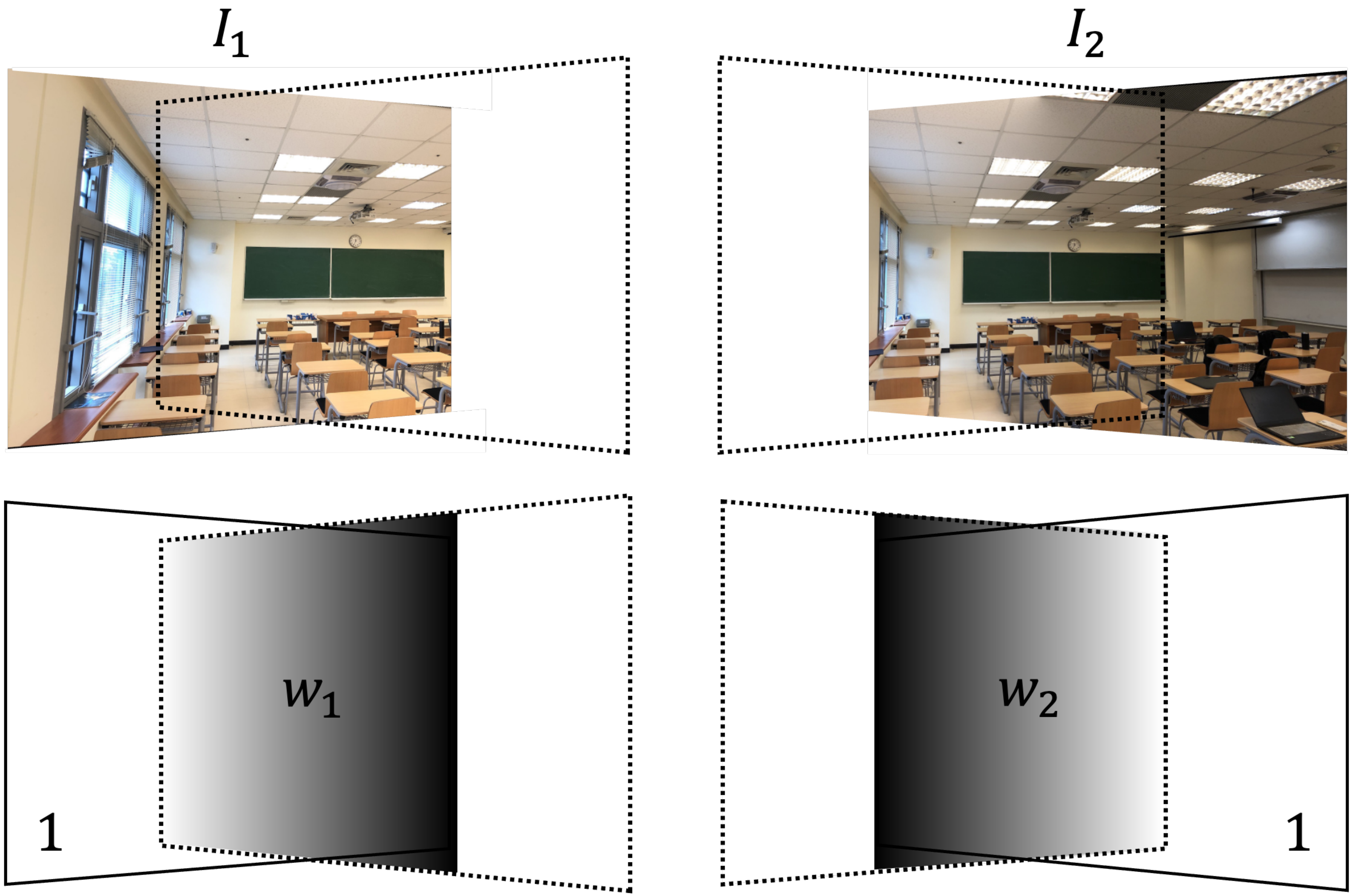


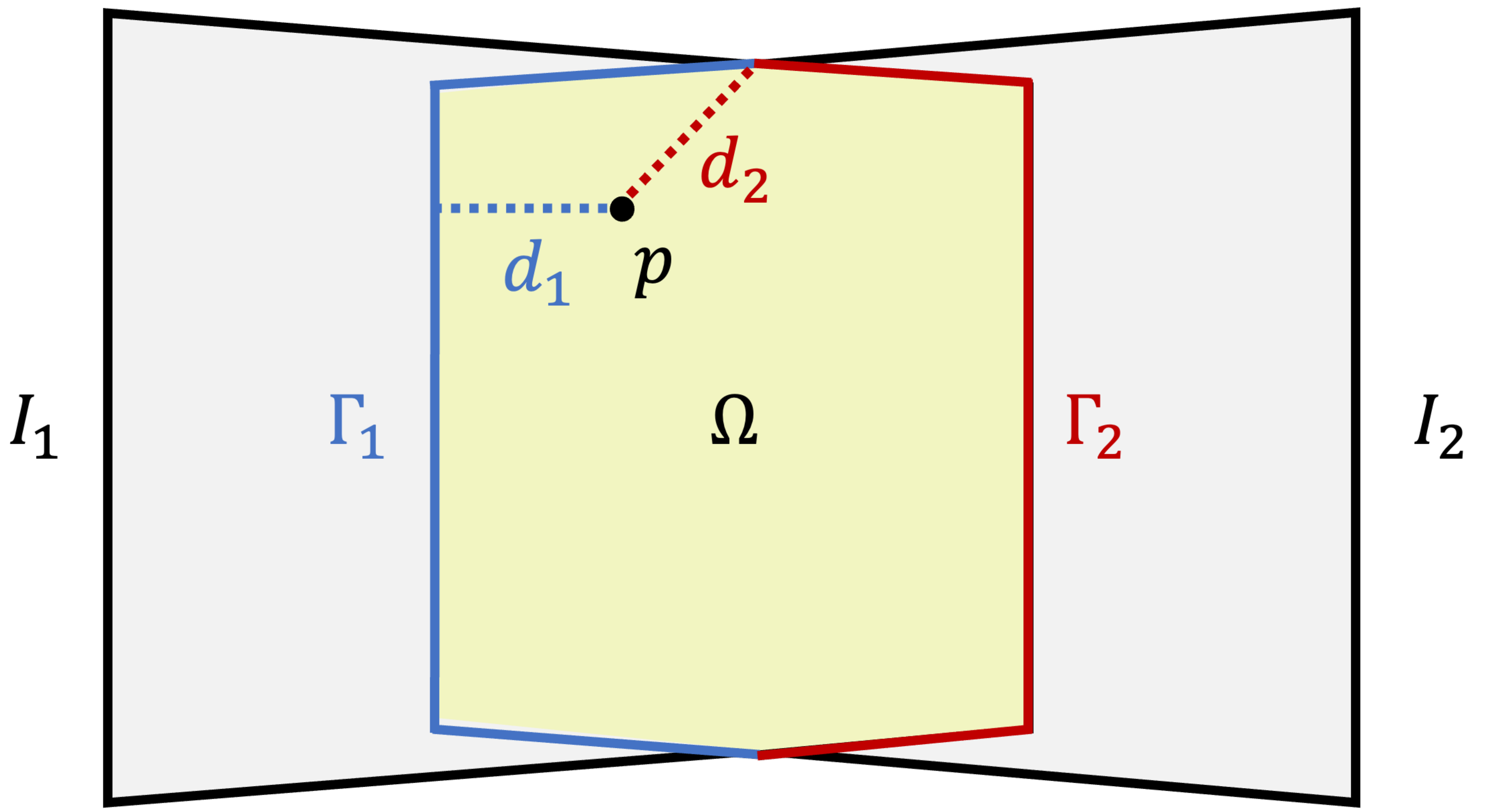
Image Blending: Continuous Mask

Define image mask functions $w_1, w_2 : \Omega \to [0, 1]$ as:

$$w_1(p) = \frac{d_2(p)}{d_1(p) + d_2(p)},$$
$$w_2(p) = \frac{d_1(p)}{d_1(p) + d_2(p)},$$

where

$$d_1(p) = \min_{p \in \Omega}\{\|p - q\| \mid q \in \Gamma_1\},$$
$$d_2(p) = \min_{p \in \Omega}\{\|p - q\| \mid q \in \Gamma_2\}.$$



The blended image I' is then calculated as:

$$I'(p) = w_1(p) \cdot I_1(p) + w_2(p) \cdot I_2(p).$$

Result of Image Blending with Continuous Mask

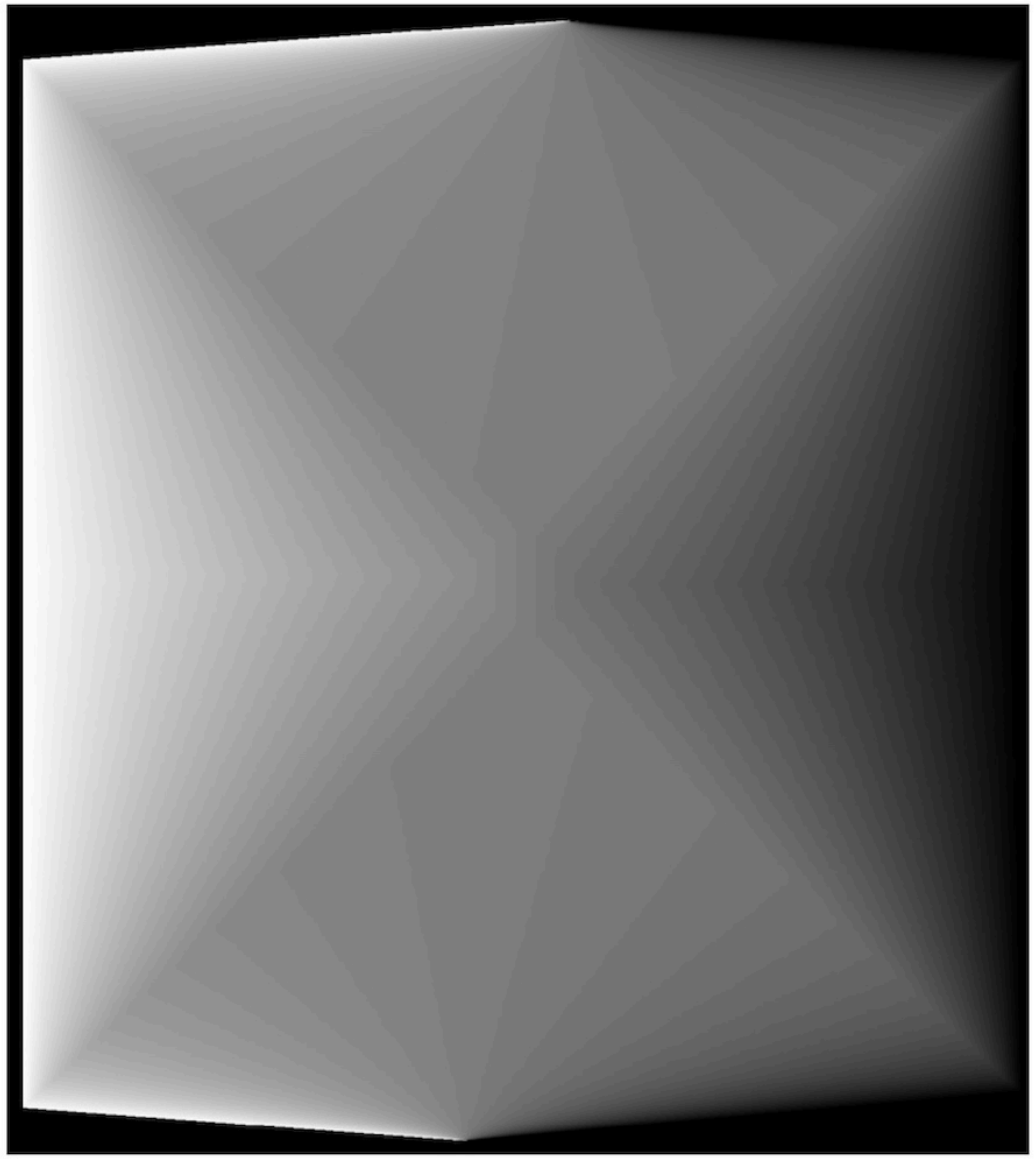
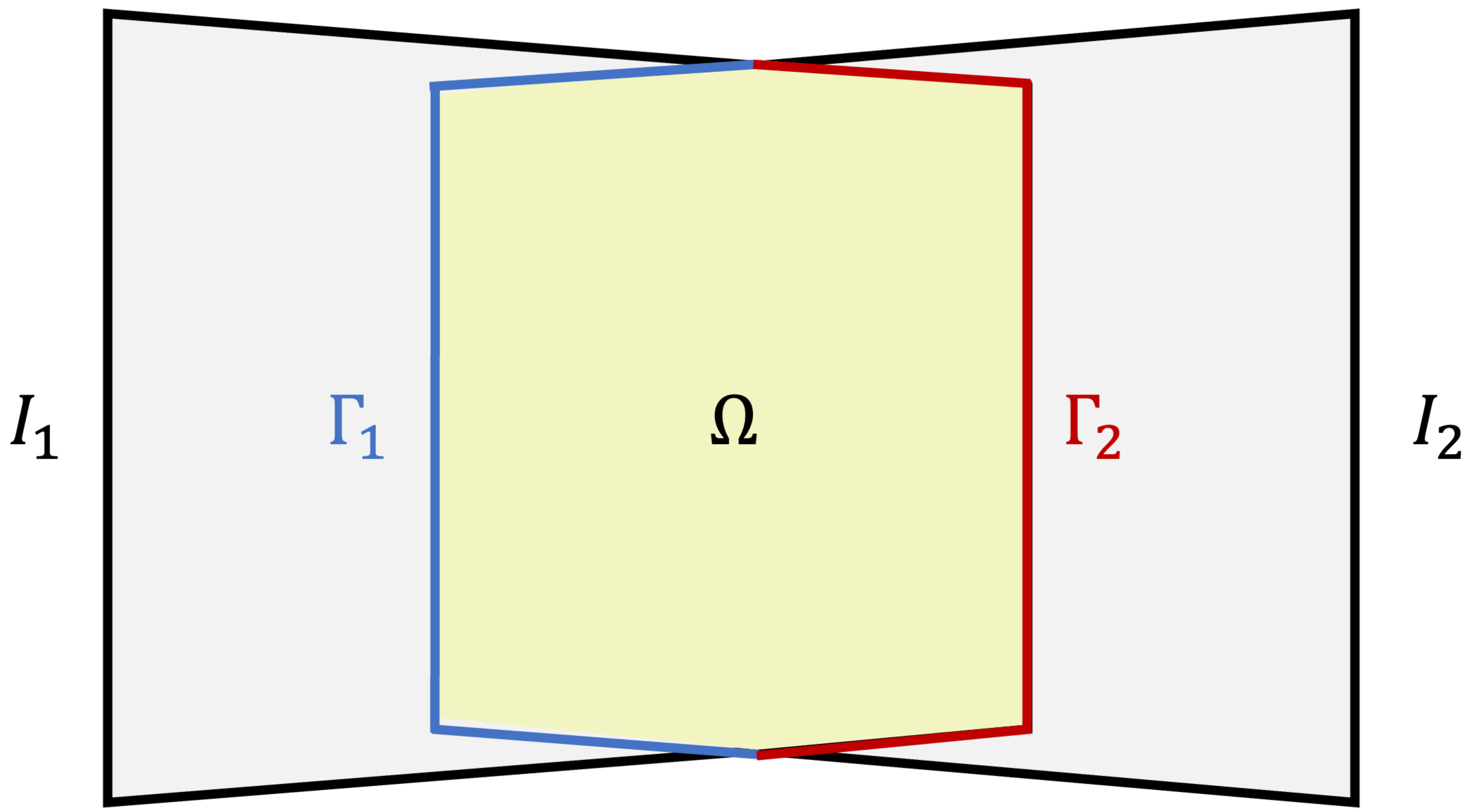


Image Blending: Poisson Equation

Define $w : \Omega \to [0,1]$ as the image mask function. The smooth blending function is derived from the Poisson equation under specified boundary conditions:

$$\begin{cases} \Delta w = 0 \text{ in } \Omega \setminus (\Gamma_1 \cup \Gamma_2), \\ w\,|_{\Gamma_1} = 1,\ w\,|_{\Gamma_2} = 0. \end{cases}$$



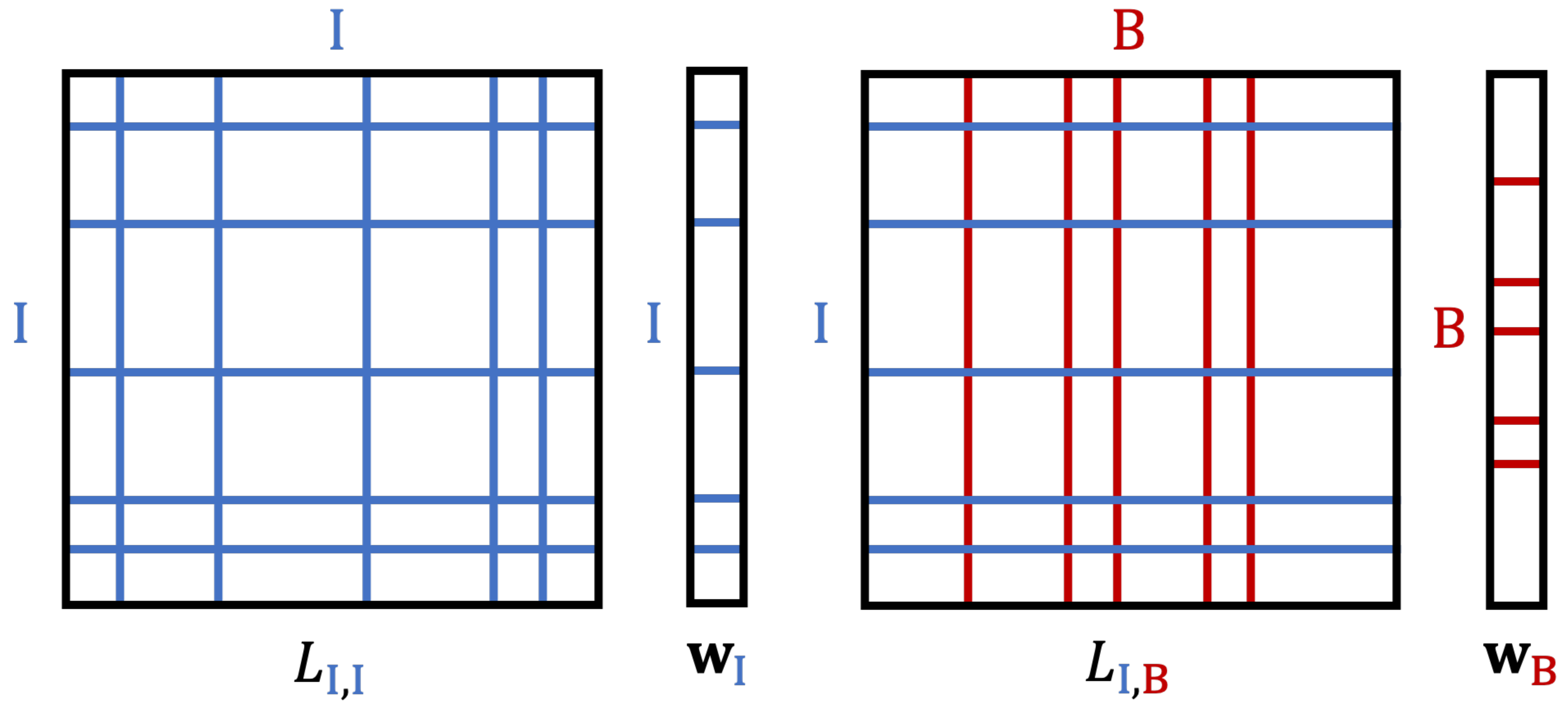
The blended image I' is then calculated as:

$$I' = w \cdot I_1 + (1 - w) \cdot I_2.$$

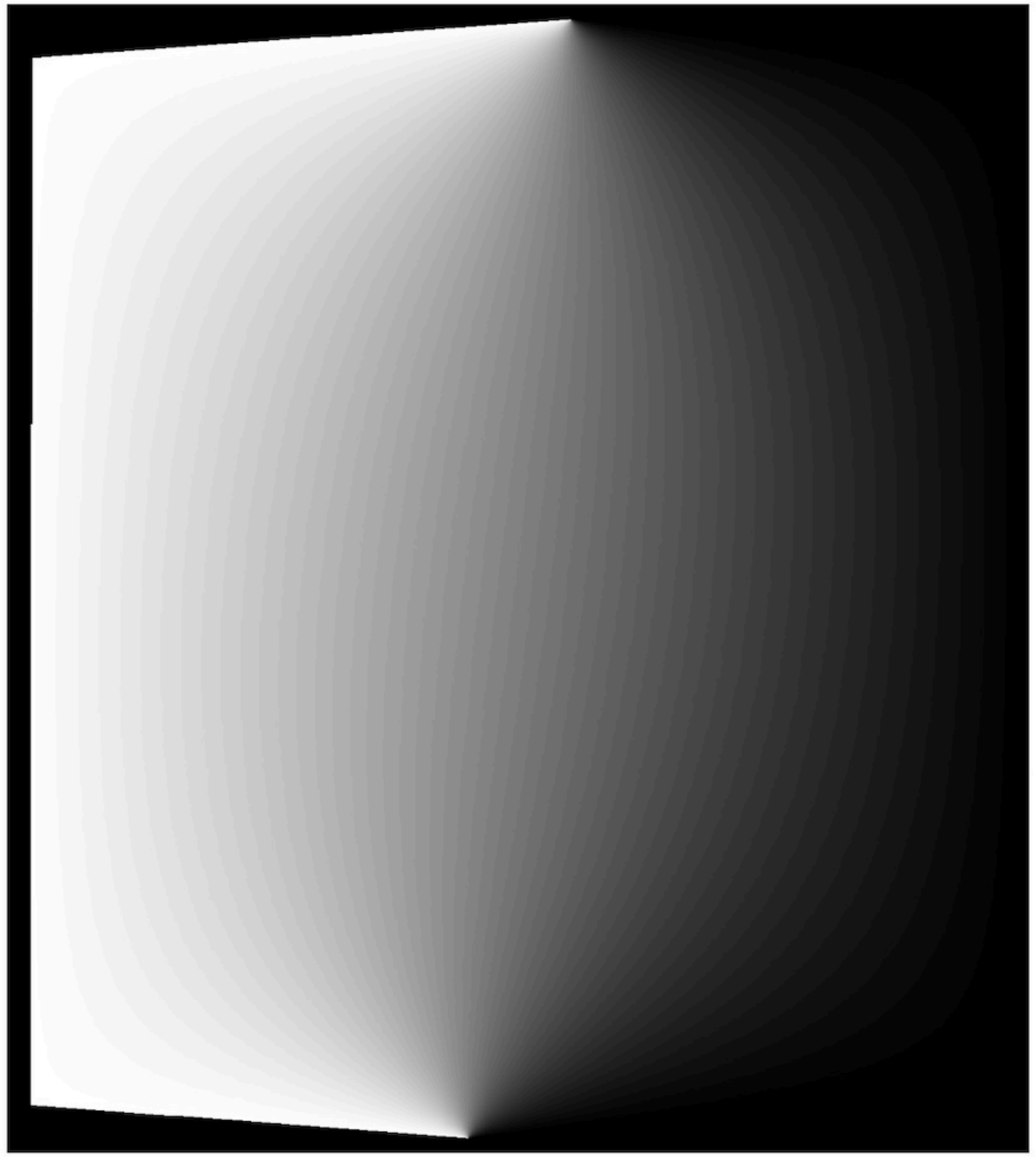
Image Blending: Poisson Equation

Let L be the Lattice Laplacian matrix and $\mathbf{w}$ be the weight vector. Segregate the point indices into interior I and boundary B. The Poisson's equation can express to the following linear system:

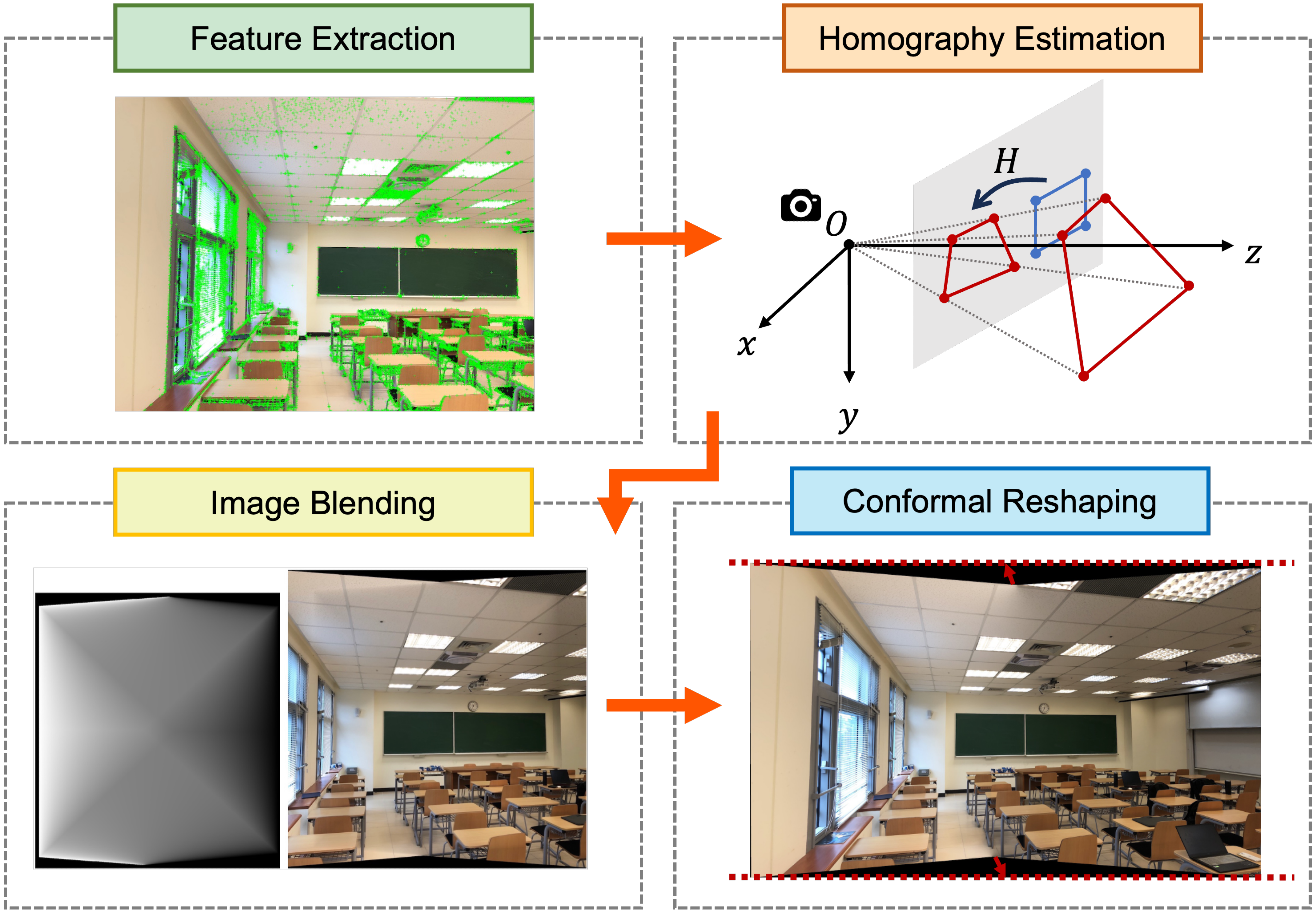
$$L_{\mathrm{I},\mathrm{I}}\mathbf{w}_{\mathrm{I}} = -L_{\mathrm{I},\mathrm{B}}\mathbf{w}_{\mathrm{B}},$$



Result of Poisson Image Blending



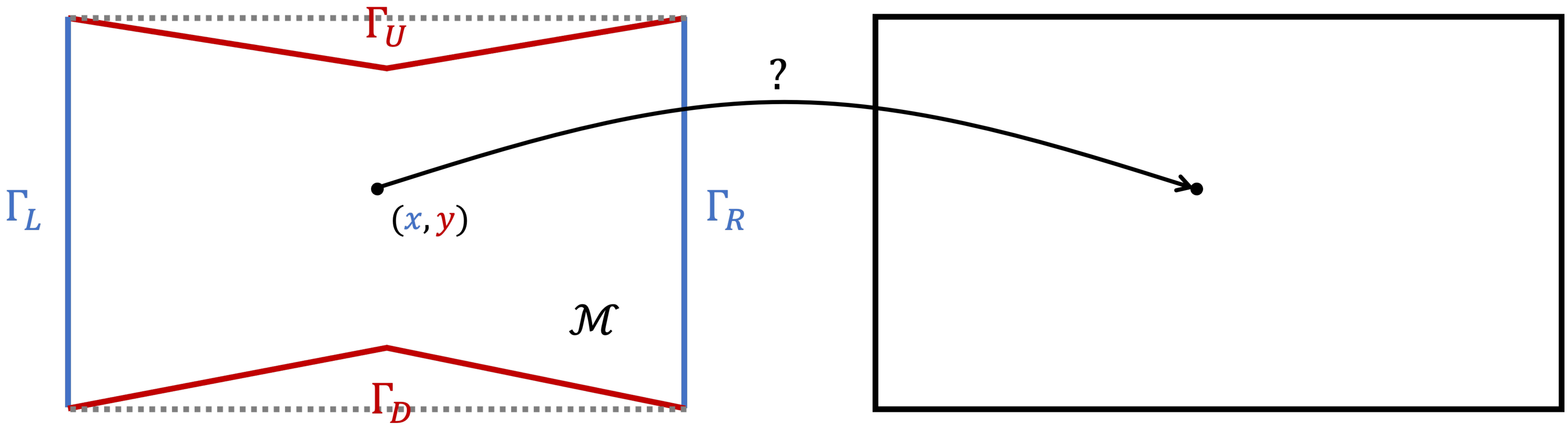
Conformal Reshaping



Conformal Reshaping

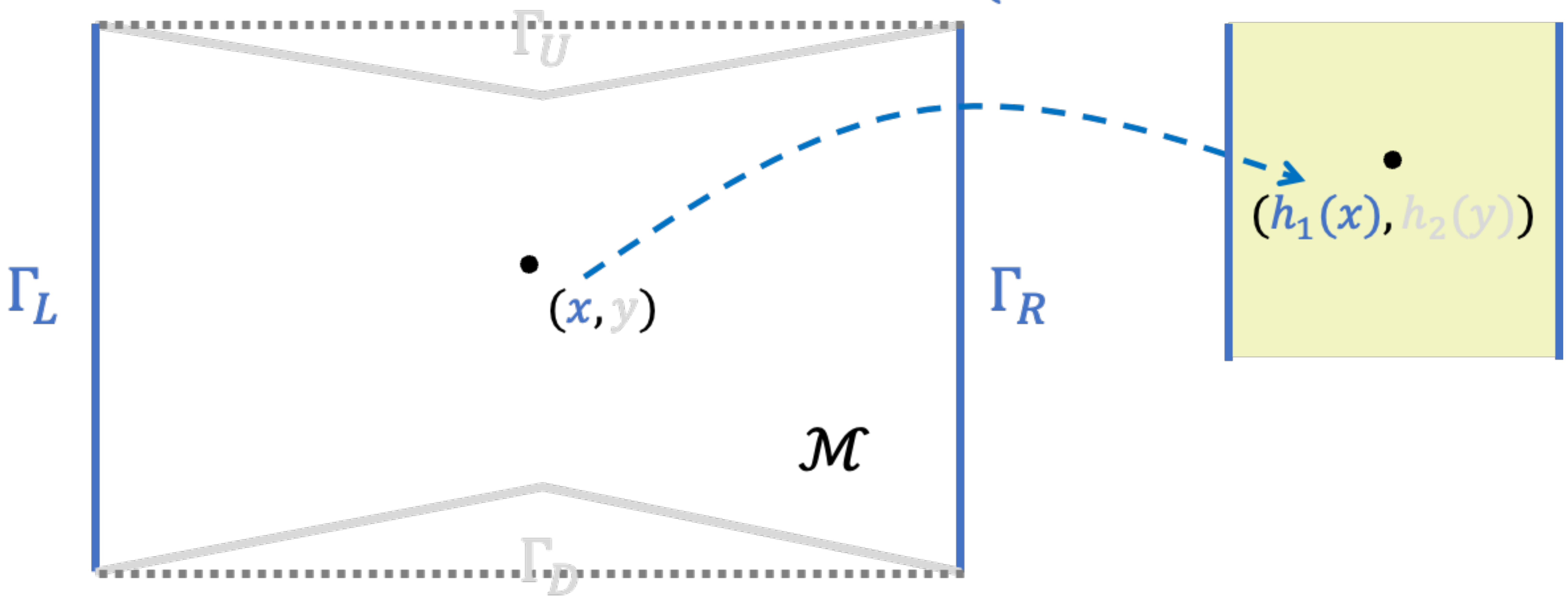
Conformal Reshaping

Construct a Delaunay triangular mesh $\mathcal{M} = (V, F)$. Our goal is to find the map $f : \mathcal{M} \to \mathbb{R}^2$ with minimum angle distortion.



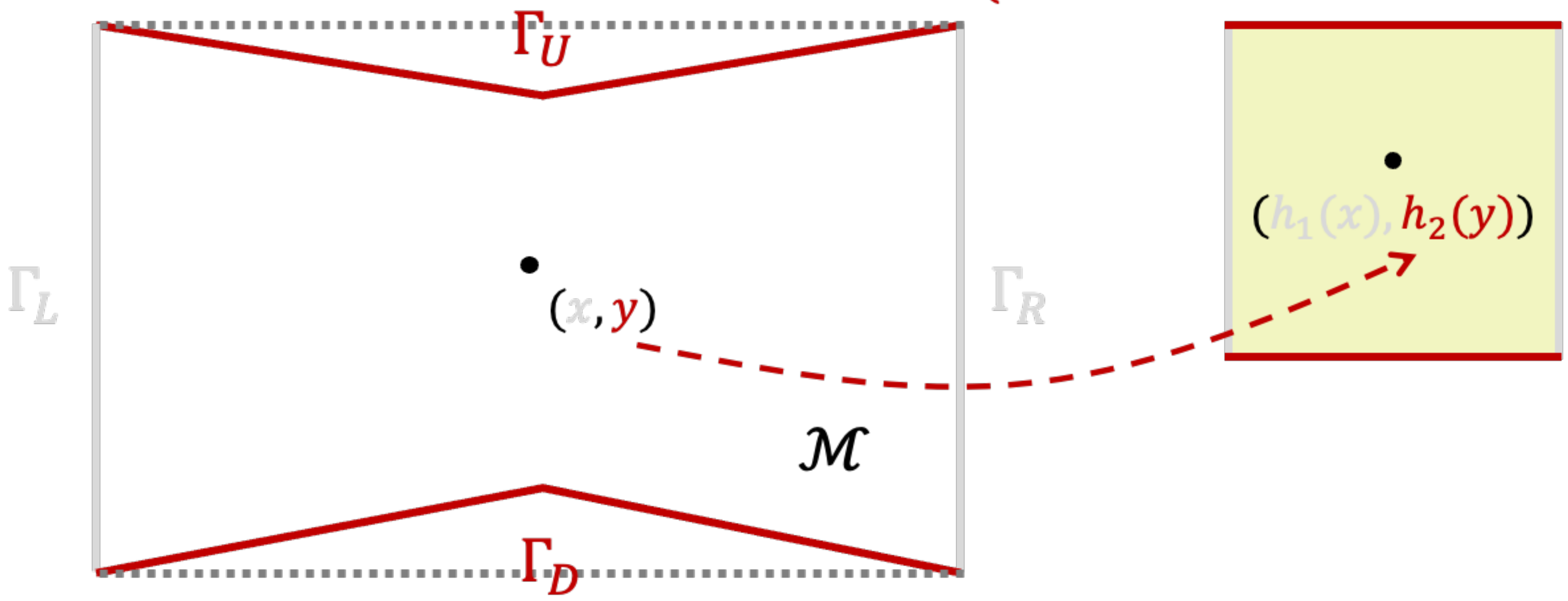
Conformal Reshaping

$$\begin{cases} \Delta_{\mathcal{M}} h_1 = 0 \\ h_1|_{\Gamma_L} = 0 \\ h_1|_{\Gamma_R} = 1 \end{cases}$$

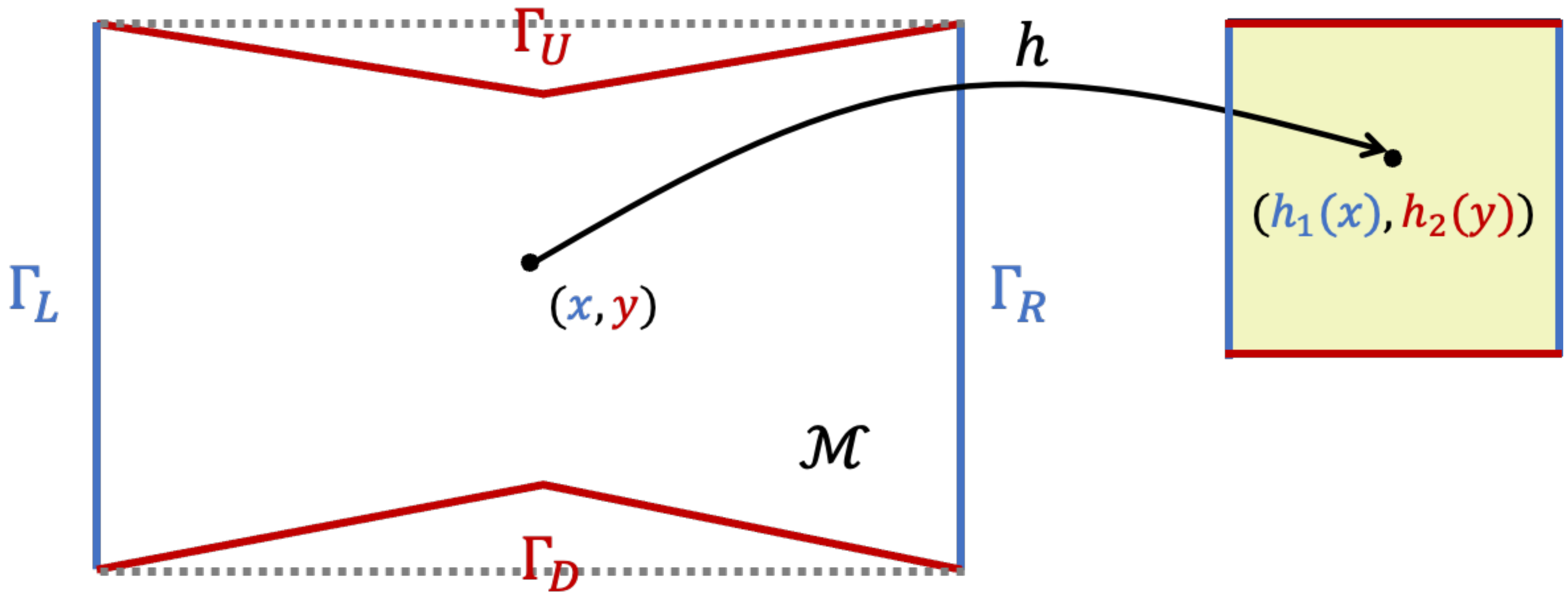


Conformal Reshaping

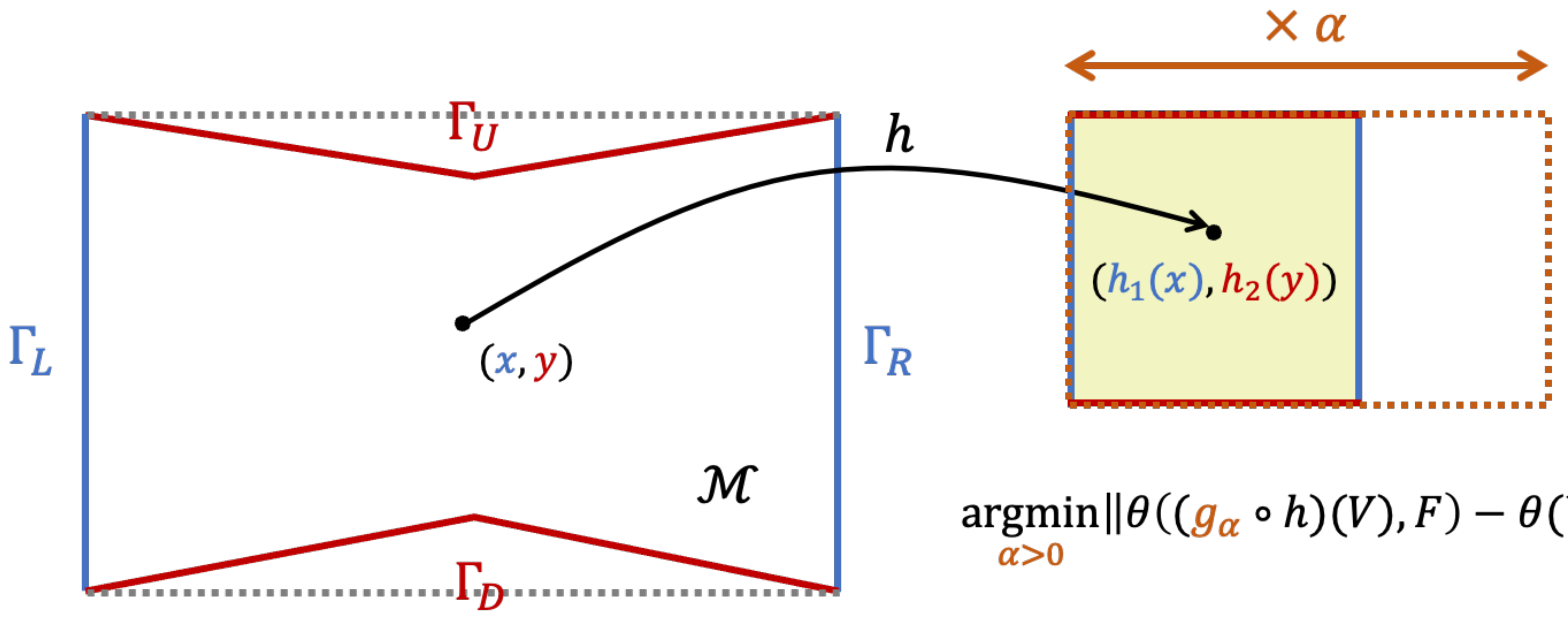
$$\begin{cases} \Delta_{\mathcal{M}} h_2 = 0 \\ h_2|_{\Gamma_D} = 0 \\ h_2|_{\Gamma_U} = 1 \end{cases}$$



Conformal Reshaping

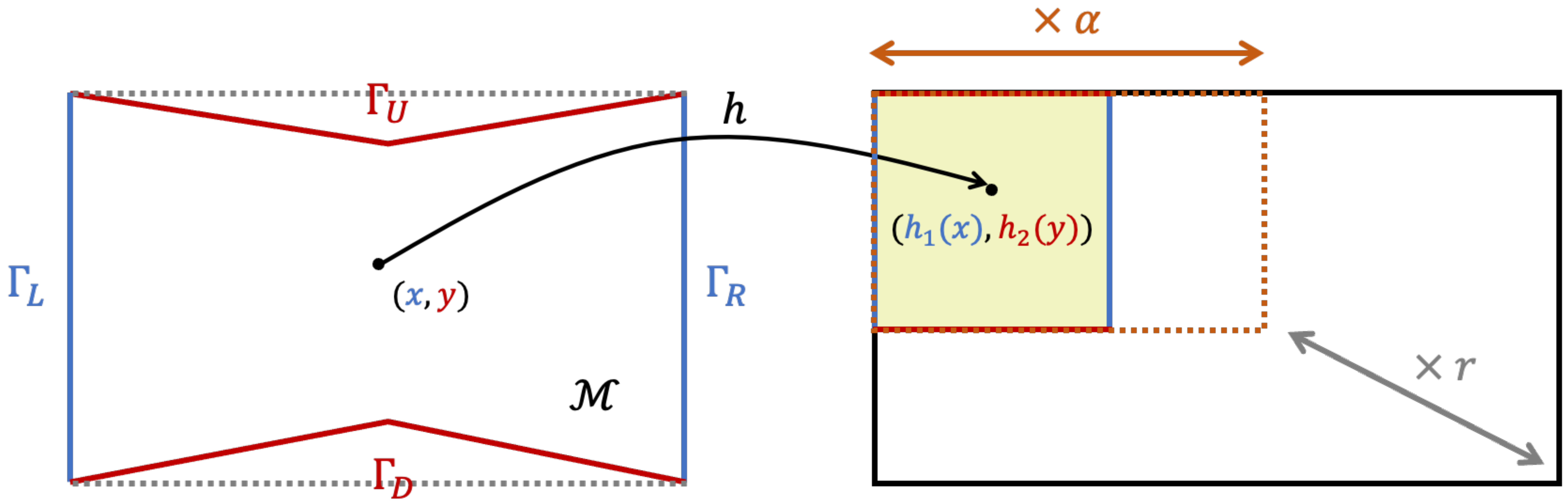


Conformal Reshaping



$$\underset{\alpha>0}{\operatorname{argmin}} \|\theta((g_\alpha \circ h)(V), F) - \theta(V, F)\|_1$$

Conformal Reshaping



Conformal Reshaping Algorithm

Given the triangular mesh $\mathcal{M} = (V, F)$ and height scale factor $r > 0$.

1. Compute harmonic mappings h_1, h_2 solving Poisson equations:

$$\begin{cases} \Delta_{\mathcal{M}} h_1 = 0, \\ h_1 \mid_{\Gamma_L} = 0, \\ h_1 \mid_{\Gamma_R} = 1, \end{cases} \quad \text{and} \quad \begin{cases} \Delta_{\mathcal{M}} h_2 = 0, \\ h_2 \mid_{\Gamma_D} = 0, \\ h_2 \mid_{\Gamma_U} = 1, \end{cases}$$

2. Determine optimal aspect ratio α to minimize angular distortion:

$$\alpha^* = \operatorname*{argmin}_{\alpha > 0} \|\theta((g_\alpha \circ h)(V), F) - \theta(V, F)\|_1.$$

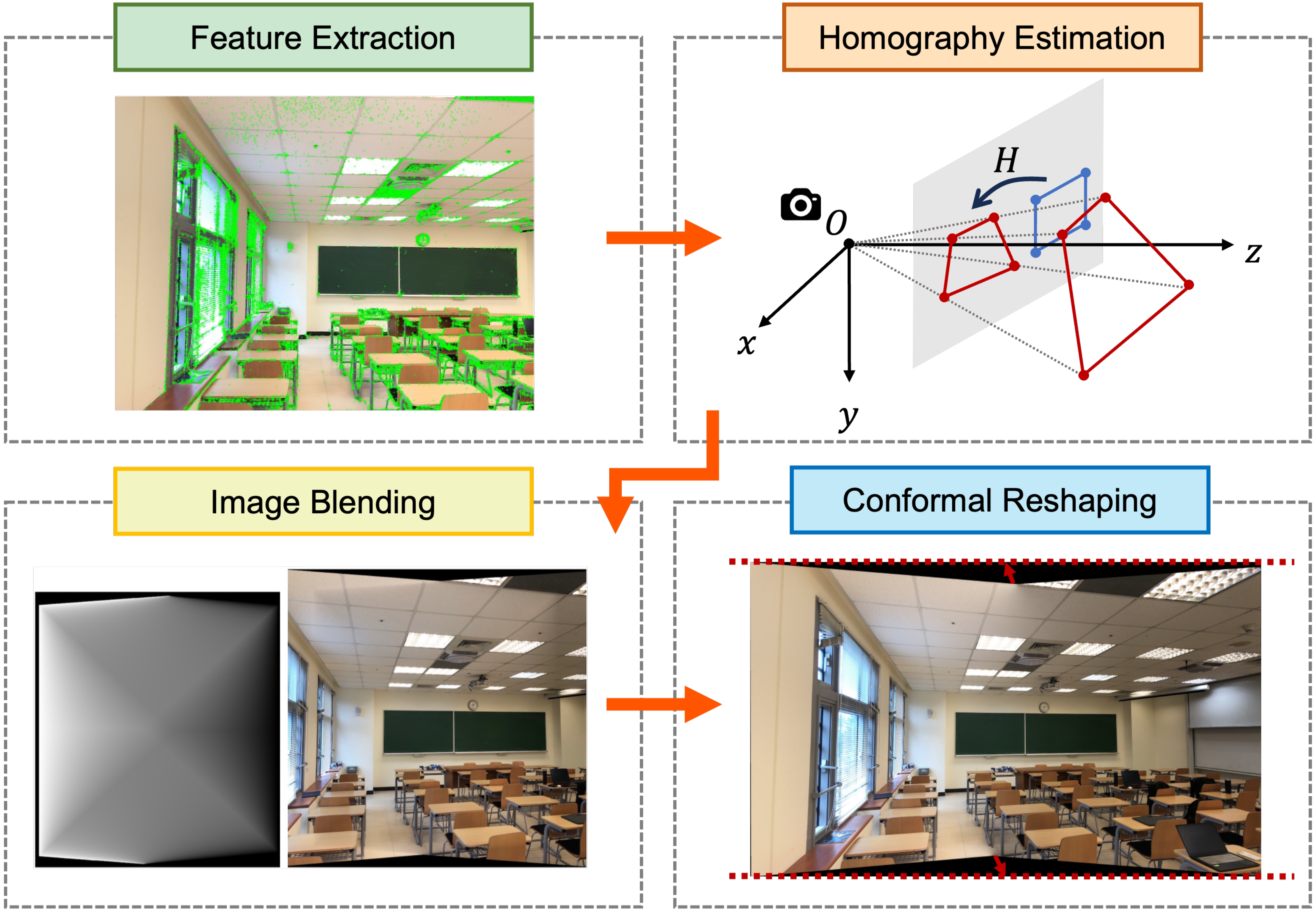
where $g_\alpha(x, y) = (\alpha r x, r y)$.

Result of Conformal Reshaping

(a) Stitched image w/o conformal reshaping.

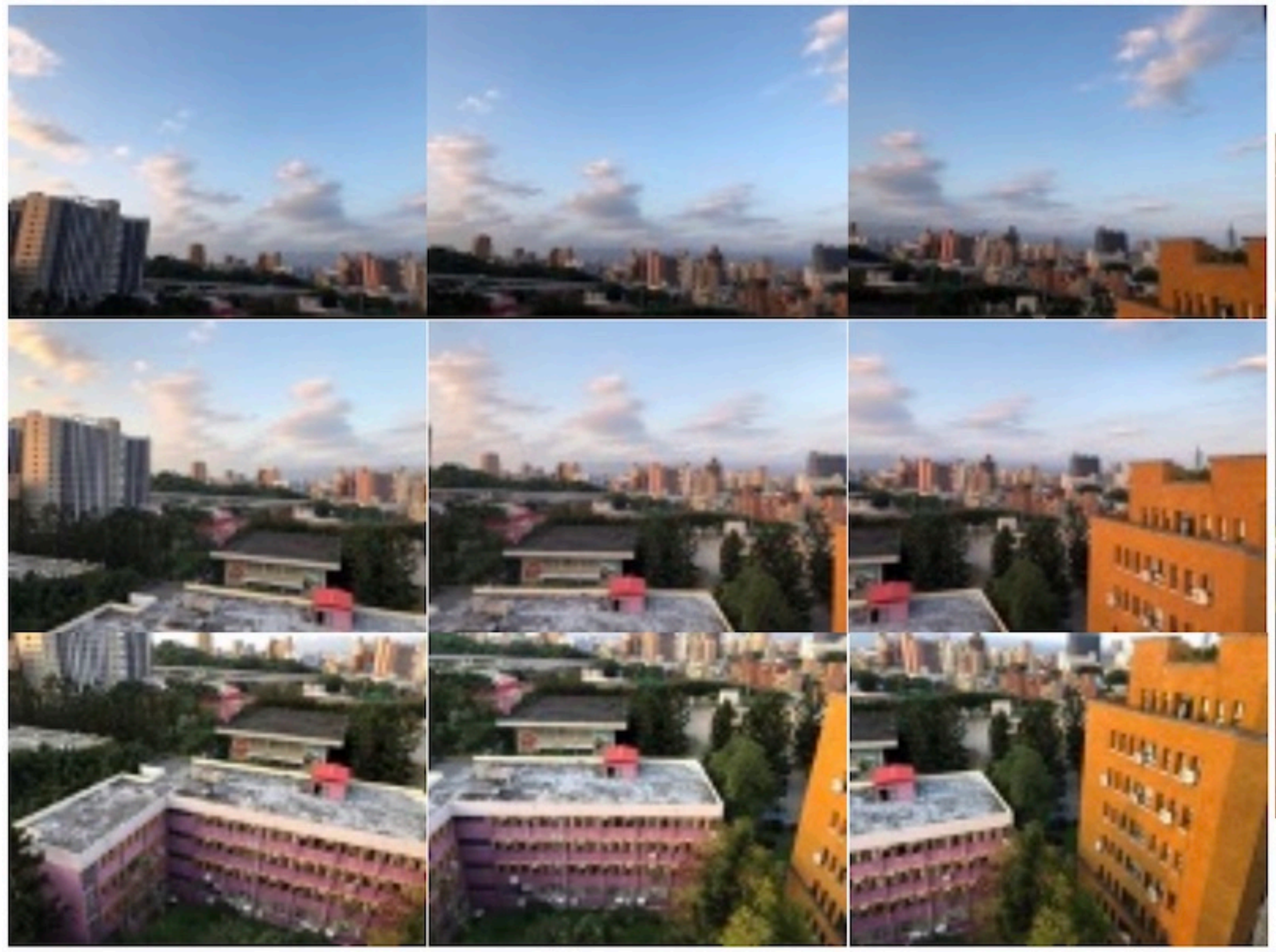
(b) Stitched image w/ conformal reshaping..

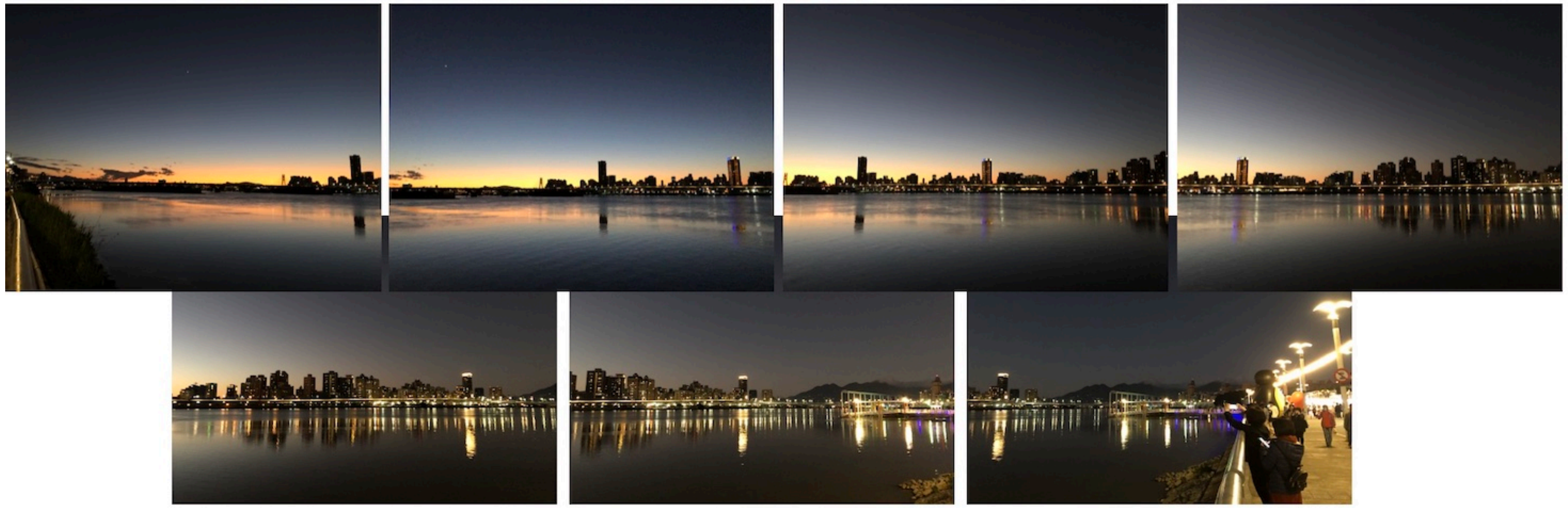
End-to-End Image Stitching Pipeline



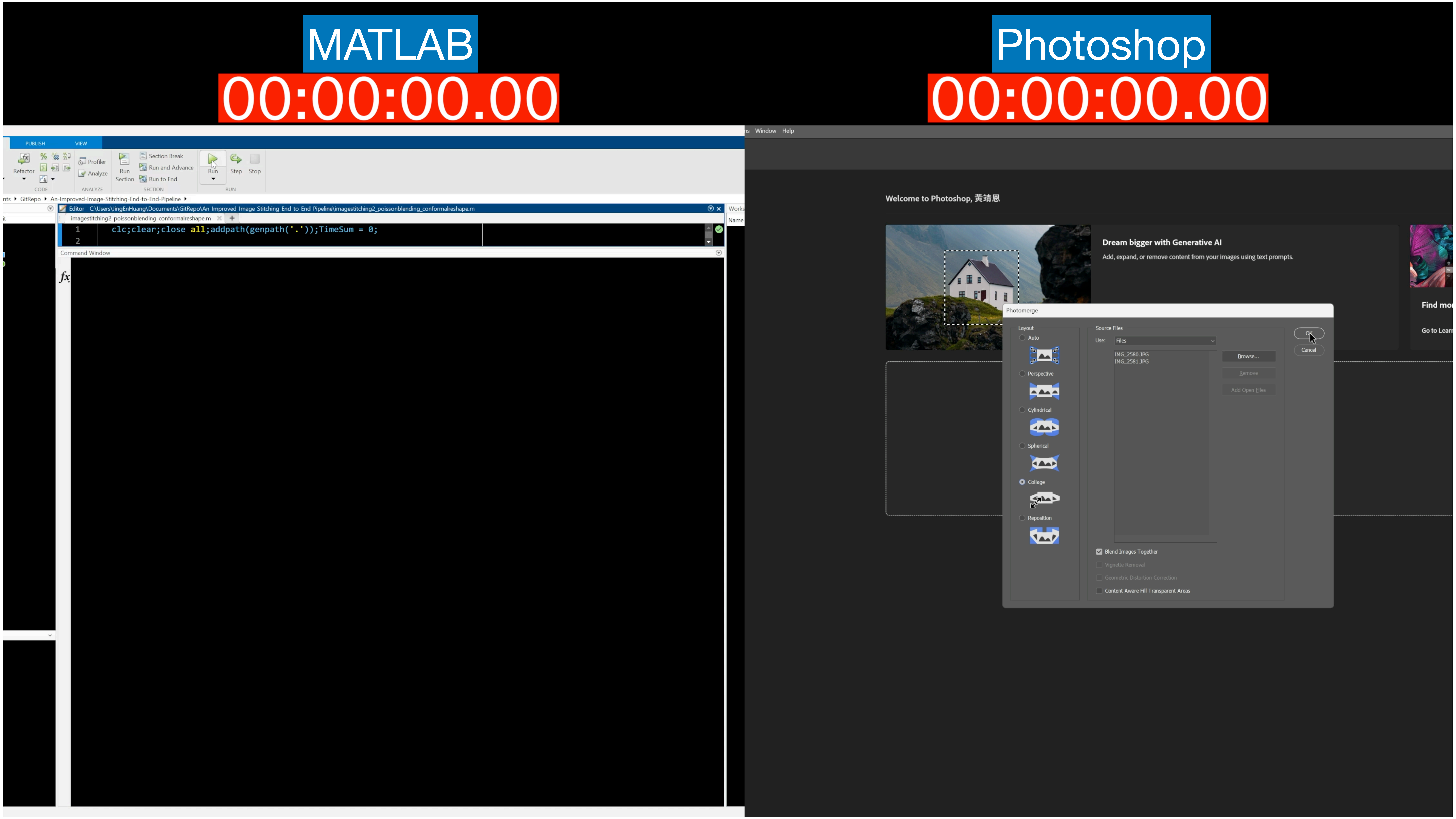
Perfect Results

Perfect Results

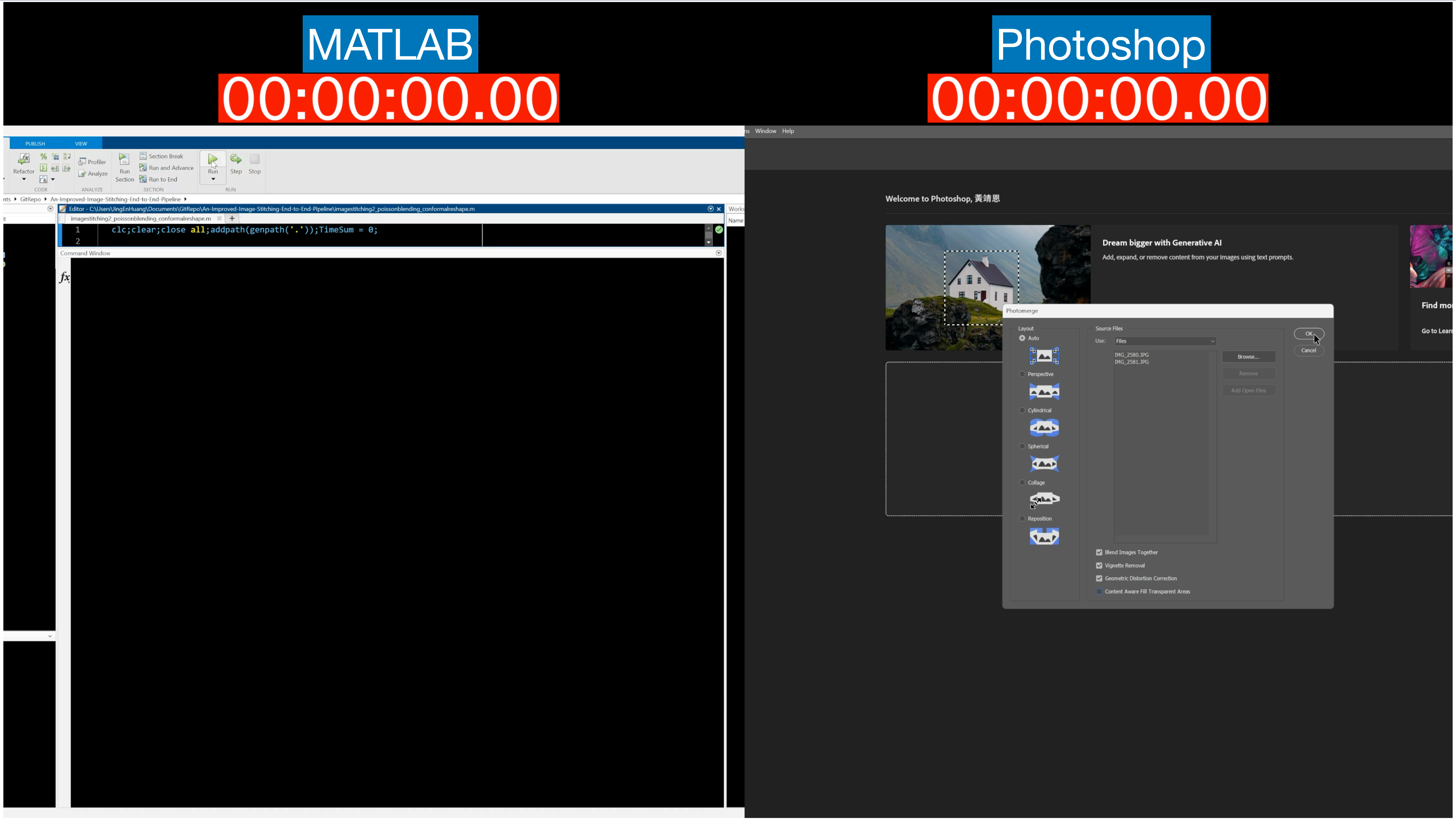




Rapid Results of Comparing to Photoshop (I)



Rapid Results of Comparing to Photoshop (II)



Take Away

- Scientific Computing enables robust and accelerated solutions for linear systems.
- Employing strategic techniques enhances the accuracy of image stitching.
- We developed an end-to-end system for perfectly and rapidly stitching images.

Thank you!